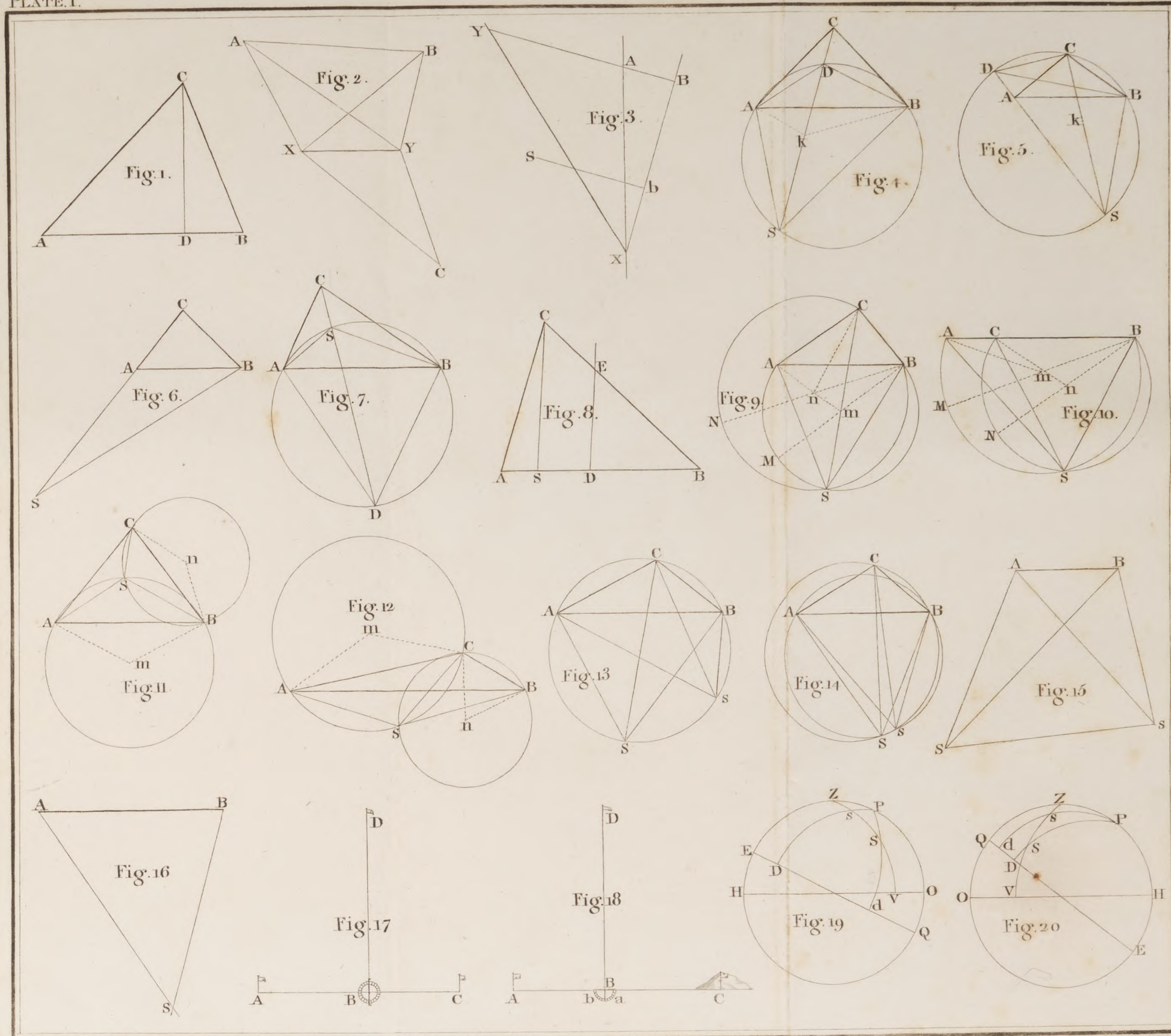
PLATE. I.



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J. Walker Sculp.

A

TREATISE

ON

MARINE SURVEYING.

In Two Parts:

By MURDOCH MACKENZIE, Sen.

Late Marine Surveyor in his Majesty's Service.

CORRECTED AND REPUBLISHED, WITH A SUPPLEMENT,

By JAMES HORSBURGH, F. R. S.

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PREFACE.

THIS Treatise on Marine Surveying, by the late Mr. Murdoch Mackenzie, allowed to be the most scientific, useful, and exemplary work, on that branch of nautical knowledge, ever published in this, or perhaps in any other country, has nevertheless not been re-printed since its appearance in 1774.

To those, therefore, who are anxious to reach the summit of nautical science, and to naval officers in general, a republication of this valuable work, will no doubt be very acceptable. Although the original work has been preserved in the form given it by the author as nearly as possible, some alterations were necessary, on account of the rapid improvements navigation has lately received, by the introduction of marine chronometers, and other means.

A supplement has also been added, containing some interesting examples, with precepts relative to marine surveying, and other information applicable to the advancement of young officers in useful knowledge.

TO THE RIGHT HONOURABLE

SIR JOSEPH BANKS, Bart. K.B. P.R.S.

&c. &c. &c.

SIR,

WITH sentiments of the highest satisfaction, I avail myself of this opportunity to testify the respect and veneration, which I feel towards you, as the universally acknowledged patron of science, and of every invention or improvement calculated to advance the happiness, and to promote the welfare of mankind.

In the humble exertions which I have used to encrease the safety of navigation, by cultivating the science of nautical geography, I have derived great encouragement from your enlightened countenance and support; and the reflection that those exertions have been honoured with your approbation, will always afford me the most lively gratification.

Your

Your opinion of this Treatise on Marine Surveying, has encouraged me to hope that it will be found of public utility; and with it, I desire to record my strong and grateful sense of all the favours which you have been pleased to confer upon me.

I have the Honour to be,

With unfeigned Respect,

SIR,

Your most faithful,

and greatly obliged Servant,

JAMES HORSBURGH.

HYDROGRAPHICAL OFFICE,
EAST INDIA HOUSE.
1st JAN. 1819.

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INTRODUCTION,

Or, a Preliminary Essay on

DRAUGHTS AND SURVEYS

OF THE

SEA-COAST.

NO branch of practical geometry has been so little considered by men of science, as marine surveying: it has not been systematically treated of, by any author, seldom taught by a master, nor have surveyors given publicity to their operations and methods of surveying. To this reserve of writers, and silence of practitioners, it may be ascribed, that an art of great importance in navigation has hitherto received little improvement; that in practice little or no distinction is made between land and coast-surveying, though they differ essentially from each other in their nature and circumstances; that so few draughts are found to answer the end proposed; and that the merits of the draughts to which seamen trust their lives and fortunes, are seldom judged of by any other rules than the *recency* of the publication, the *neatness* of the engraving, the Authority under which the survey was executed; or,

sometimes, by the rank and reputation of the persons to whom they are inscribed. Had coast-surveying been considered with the attention it deserves, the various sorts of draughts often indiscriminately made use of at sea, would, before now, have been characteristically distinguished from one another; the particulars of which a complete draught consists would have been sufficiently known; the several methods of surveying compared together, and the most perfect manner pointed out and explained. Whoever undertakes a survey of the sea-coast, is under the necessity of devising a plan of procedure for himself, and performing most of the operations by his own ingenuity and skill in the principles, without the advantage of the speculations and experience of others to assist him. This want of previous instruction, and the difficulty and trouble of frequent investigation, seem to be the *chief* reasons, that coast-surveyors commonly follow the method which their antecedent business or applications suggest, rather than that which a thorough knowledge of the theory, and strict attention to the nature and circumstances of the coast would have directed. The navigator, or experienced seaman, constructs his charts chiefly from sea-journals of courses and distances: the engineer and land-measurer, by a continued mensuration of sides and angles; or by a connected chain of triangles. Some measure their distances with a wheel, some with a chain, and others with a rope: one takes his angles with a needle, one with a theodolite, and a third with a common compass: the way in which one proceeds in a survey, necessarily restricts his performance within narrow bounds: another takes a method, by which he surveys more coast in one year, than he could travel over in three; surveying a little himself, and copying a

great deal from others, without distinguishing the *certain* from the *uncertain*.

Wherever the safety of shipping is concerned, the public has a right to some satisfaction with respect to the nature and grounds of the publication. It is not sufficient to say, in the title, that it is an *actual* survey; or, a *new* and *accurate* survey: it ought to be accompanied with, at least, a short account of the *fundamental operations*, and *manner* in which the survey was conducted. This would enable seamen to estimate the merits of the charts they were trusting their lives to, and to judge whether they might be depended on, and in what circumstances this would be imprudent. A faithful answer to the following questions would be sufficient for this purpose, and might be easily subjoined to the title of any chart.

1. *How long was the fundamental base-line? where, and how, was it measured?*

2. *In what manner was the survey carried on from that base-line? And with what instruments were the angles taken?*

Or, if charts were classed under different heads, with epithets, or appellations, expressive of the manner in which each was done, and published with that epithet, or under that appellation only; this would in a great measure answer the same end, and point out readily the nature and peculiarities of each. Such a capitulary is attempted in the following enumeration of draughts and surveys; and will serve to explain what is here proposed.

I. A General, or Nautical Chart, is commonly constructed and drawn by some experienced seaman from his own observations and journals of the

courses and distances from head-land to head-land, and point to point; with the intermediate spaces filled up from such charts or maps as are at hand, or are in most repute. Sometimes on the margin of the chart are added larger sketches of some of the most noted harbours he is acquainted with, drawn from his own memory, or copied from others. These charts are always graduated by a very small scale, so as to comprehend a large extent of land and sea. Rocks and shoals, are seldom, or never, particularly examined, but laid down according to their apparent distance from a noted head-land, or by the ship's reckoning, which is looked upon as a sufficient direction for avoiding them.

In these charts, as the distances are either usually taken by the eye, the log-line, or the ship's traverse, without any certain allowance for the influence of tides and currents;* and the courses, for the most part, deduced from the reckoning, or sometimes taken at sea by a compass; great exactness is not to be expected: and as the scale is very small, no part of the coast can be correctly delineated in the chart, to point out with certainty the situation of shoals or harbours. Notwithstanding these disadvantages, such *general charts* are necessary to navigation; and some persons take charge of ships, who have no better guide: yet, if they are not particularly acquainted with the places they are bound to, a pilot will be necessary; for such charts should not be trusted farther than for general courses in the open sea.

When these general charts are corrected by latitudes and longitudes carefully observed; and when the pub-

* These sources of error are now precluded by the application of chronometers.

lisher is so just to himself and others, as to distinguish such courses, distances, and tracts of navigation as are delineated from his own repeated observations and experience, *from* those which *he has copied*, or *taken on trust* from others; then they become very serviceable in sailing from land to land, or along an open coast; but near the shore, where the danger is greatest; and in bad weather, when a ship is under the necessity of running for a harbour, or for shelter in a bay, these general charts are of little or no service. In such circumstances, larger, and more particular charts, or a skilful pilot, are indispensibly necessary.

Charts of this sort will not be brought to their utmost perfection, till skilful persons are sent on purpose, with good instruments, to observe, on land, or near it, the latitudes and longitudes of the most remarkable points and promontories which ships have frequent occasion to approach. Distant views of any remarkable land ought also to be taken; and each observation, with its several circumstances, should be preserved and published along with the chart constructed from them. When a correct method of surveying is more generally understood by nautical men, the intermediate parts might be particularly investigated; which (excepting banks and channels that are of a variable nature) would be as permanent as the other.

II. Abbreviated Collections of Draughts, as they comprehend a great deal of coast in a small compass, are of the same nature with the nautical chart last mentioned, and can serve for no other purposes in navigation but to direct ship's courses from land to land, or from promontory to promontory along a coast.

These are commonly the performances of map-sellers or others, who perhaps never saw any of the places they delineate, and, for that reason, cannot be supposed capable of making a proper choice of the draughts they abridge: besides, in reducing a diversity of scales and draughts into one, notwithstanding the utmost care, they can hardly avoid introducing new errors, or inaccuracies, exclusive of what might have been in the originals; and the distinctness that was in the latter, will certainly be lost in the abbreviation. These miniature collections often do more prejudice than service to navigation: because, from the comparative lowness of the price, the names and specious authorities annexed to the title, many are inconsiderately led to purchase and preserve them; while the originals, that would have been of much more service, are as inconsiderately neglected and lost.

III. A Memorial Sketch, is a delineation of a harbour, or any part of a coast, from the memory only; without notes, or immediate sight.

When such a sketch is made by one who has been often in the place, or who has viewed it with particular attention, it may convey a general idea of a bay, harbour, or island, fit to gratify curiosity, or enlarge geography, by shewing that some such places are there: but as every part of the sketch was at first by guess, and that guess delineated by recollection only, the whole must be very uncertain, and of little service to shipping; farther than to admonish sailors to keep a look-out, when they sail that way.

IV. An Eye-Sketch, is a delineation of any harbour, or part of a coast, done by the eye at one station,

without measuring distances; and drawn according to the apparent shape and dimensions of the land.

Though here, neither bearings, distances, the figure of the land, nor the position of rocks or shoals, can be exact; yet a good draughtsman, may make such a sketch convey a sufficient notion of a small bay, harbour, or island, to be serviceable on *some occasions;* and in moderate weather, with the help of the lead, to direct a vessel to the safest channel and ordinary anchorage. The accuracy of such a draught, depends on chusing an advantageous station, so that the principal parts of the harbour, or bay, may not be too far from the eye; for the remotest parts will always be most unlike. It is likewise proper, to chuse the station near the sea, on that side of the bay, or harbour, next to the common channel; because, remarkable parts on that side of the bay and channel, may then be delineated according to their bearings, by a compass. Such a sketch is soon made by one moderately skilled in drawing: and if skilful officers, in places unknown, or not surveyed, did take such sketches, it would be of some service both to navigation and geography.

V. An Ambulatory Draught, is made by walking along the shore, taking the bearings from point to point with a compass, estimating their distances by the eye, and sketching the figure of the coast between them. If to this, remarkable objects on land are added, and such rocks, shoals, and ledges as are visible at low water, or by breakers, with the depths in the principal channel, and in the anchorage, the draught will be a distinct representation of any single bay, or harbour, and prove serviceable to vessels on several occasions. But if it is continued for some

leagues on an irregular coast, the erroneous distances will affect the bearings, and also the figure and dimensions of the whole draught: gross errors in the more considerable parts must then be corrected by making the less considerable parts more erroneous; till at length the whole will become too evidently distorted and out of proportion to admit of farther alterations; and then of necessity, the survey must be discontinued.*

When such draughts are confined to small parts of a coast, or to single bays, or harbours, the errors in them are less sensible; and they become serviceable by exhibiting plainly what kind of bays and harbours are to be met with on that coast: and, when rocks, shoals, and soundings are inserted, they will be found a pretty good direction for vessels that may happen to fall in with them, provided at the same time they are sure what harbour it is which they have fallen in with: but to strangers, who are not well acquainted with the land, separate draughts of harbours will be found of little use, especially in bad weather, when a sure direction is most desirable; because of the difficulty of finding any particular harbour, and the danger of a mistake in running for it. A ship must be off the mouth of the harbour before such a draught can be of use; and the pilot who brings her there, may likewise carry her in.

When a person makes a draught of a bay, or harbour in this manner, at the time he goes along the first side delineating it, the magnetic direction of the different points and heads on the other side, should be drawn out on the paper, and their positions limited to that di-

* It is likely that most of the particular draughts formerly used by seamen have been made in this manner.

rection, which will render the bearings right,† however erroneous the distances may be: if this is neglected, both the distances and opposite bearings will be false, and the draught unlike in every respect.

None of the three last-mentioned performances ought to go under the name of surveys, because there is no mensuration of distances in them, nor any thing determined in a geometrical manner.

VI. A Disjunct Survey, is, when the harbours, bays, or islands in any country, are each surveyed separately in a geometrical manner. Or, when one, or more, straight lines are measured on a level plane in each harbour, bay, or island; the angles taken with a good instrument, and the distances of the several places determined from thence trigonometrically.

In this survey, all the rocks, shoals, and channels are supposed to be carefully examined, and distinctly delineated, by a large scale; and the buoys, beacons, landmarks, remarkable hills, groves of trees, churches and buildings, necessary for directing vessels on any occasion, inserted in their proper places and miniature forms; the various diversities of the sea-coast, and of the rocks, shoals, and sand-banks along it, represented so as these varieties may be readily known by inspecting the draught. Such a draught will be a minute and exact representation of each place, as far as relates to shipping; and is, or ought to be, the most complete of any, within the limits of the bay, harbour, or island surveyed. There is, however, a material defect in draughts of this sort, viz. rocks, shoals, and sand-banks that lie off the harbour, or coast, can seldom be laid down either in their

† If the needle be not drawn aside from the magnetic meridian, by local causes, or metallic attraction.

exact dimensions, or in their proper places; nor can intersecting bearings be given for finding them out; nor land-marks for avoiding them on all sides: because, each draught comprehends so small an extent of coast, objects properly situated for these purposes, cannot be included. Detached draughts of small extent, although accurate, will not be found of much more service to navigation, than the ambulatory draughts before-mentioned: because, neither of them are of use till the mouth of the harbour is closely approached; when the apparent similitude between the ambulatory draught and the land, will guide a ship to the proper anchorage, as well as an exact similarity in the geometrical draught; for in sailing along, the eye cannot perceive the difference. No person unacquainted with the coast, would willingly run for a harbour, by the detached draughts that are to be seen on the margins of some maps and charts. Nevertheless, disjunct draughts, when well executed, give great satisfaction, by shewing the nature and capacity of bays and harbours exactly, and are useful records to those who have been in them, and are so well acquainted as to know the land at some distance: but a stranger to the coast, can receive very little benefit by them, except that they may prove a check on unskilful pilots.

VII. A LINEÄNGULAR SURVEY, is, when the coast is measured all along with a chain, or wheel, and the angles taken at each point and turn of the land, with a theodolite, or magnetic needle.

A few bays, that happen to be bordered with sand or marsh, may be suited to such operations; and then, if great care is taken, a pretty exact survey may be made. But, it is only a small part of any coast that is level and smooth enough to admit of a continued mensuration with

any tolerable degree of accuracy. And, when it is considered, that every error, or inaccuracy, in any one measured distance, occasions a proportional error in all the other subsequent distances; that every inaccuracy in the angles, affects all the other subsequent angles and sides; that, among such a number of angles, inaccuracies, (perhaps mistakes,) are scarcely to be avoided; and difficult to discover, so as to be properly corrected. When these things are considered, it will not be reckoned a paradox to affirm, that an irregular coast may be surveyed by the eye and a compass, as exactly as in the lineängular manner. A surveyor will easily be satisfied of this, if he has ever tried to survey an irregular field, or small island, by measuring all the sides, and observing the angles between them: by his field-book their circumferences will never meet on the paper. Surveys taken in this way, are tedious, troublesome, and require many hands constantly employed: the distances by the draught, and by observed latitudes, differ widely from each other, and the whole will come out larger than the truth, except errors in the angles should happen to counteract the errors in the sides; and in that case, particular parts must be very much out of proportion. If the distances are measured with a wheel, instead of a chain; or the angles taken with a needle, instead of a theodolite; then the whole, and particular parts, will be still more erroneous. Lineängular surveys, continued beyond the limits of a small bay or harbour, are in no case to be depended on; and very few, even of these, can be exactly done in that manner. This, however, seems to be the most common method of geometrical surveying.

VIII. A Trigonocatenary Survey, is, when one long base-line is carefully measured on a level plane,

the angles taken with a good instrument, and from that base-line, a connected series of triangles carried on along the whole coast, by which the distances of the several heads, points, and flexures of the shore, are determined trigonometrically.

By this procedure, one source of error in the former method is avoided, viz. that which arises from a continued mensuration: but another, perhaps no less considerable, found by constant experience, to flow from a long chain of triangles all depending on one another, affects the mutual bearings and distances of the places. This manner of surveying is liable to much the same inaccuracies and imperfections as the lineängular, only not to so great a degree. When sharp objects, for forming the angular points with precision, are rarely to be met with, (which is the case on every coast) errors in angles must necessarily arise: one error produces a second, that a third, and so on; each increasing the inaccuracies of the subsequent triangles: and, after they have insensibly run through a few trigonal links of the chain, their source becomes undiscoverable; and, at length, the error of the whole, too great to be corrected in the material parts, without making very gross errors in other parts: the survey must then of necessity be either broke off, or continued with very striking imperfections.

If a coast happen to have level planes at proper distances from each other, and new base-lines be measured on these planes; and if poles, or signals, are set up forming triangles all along the coast; then, with great care, the angles may be taken, and the survey continued with sufficient exactness. But, such a procedure, would be extremely tedious; and a life-time would hardly be sufficient to survey the coast of one kingdom in this manner, with the care it requires. As planes of sufficient length,

are seldom to be found on any coast, whoever carries on an extensive survey this way, will be often under the necessity of making random alterations in the draught, at least at every new base-line, that the several parts may connect with one another, and that the whole may agree better with observed latitudes. And, after the whole is put together one way or another, the original scale must be changed, and a different one substituted in its place, better adapted to the latitudes, and to the dimensions of the whole: that is, a survey taken in this way, must be finished, before the scale that suits it is known.

IX. An Orometric Survey, is, when one long base-line is exactly measured, and the distance of the summits of two, or more, high mountains in the neighbourhood, found from thence trigonometrically: then, by angles taken on these summits, the distances of all the points, heads, turns, and other visible objects on the coast are ascertained; and from them, the positions of other intermediate objects and places. From these mountains, likewise, the distances of other mountains, farther along, are found; which serve in the same manner as the first, for determining distances along that part of the coast adjacent to them.

By this way of surveying a coast, when the foundation is carefully laid, the errors which all the fore-mentioned methods are liable to, are in a great measure guarded against: for, the error, or inaccuracy, of any one distance, or angle, is not communicated to the rest, but confined to that distance, or angle alone, or perhaps to an adjacent point or two besides, which are not of great consequence in the draught, nor affect the positions of other parts. Therefore, in a mountainous country, where the fundamental triangles are few, and the nature

of the coast otherwise favorable, a survey may be continued a great length, without any error considerable enough to affect the extent of the whole together. But shores are generally low, and of such a nature, that particular parts can seldom be perceived from the tops of two remote and high mountains; even large buildings are not often to be seen from thence: and though points and promontories, in gross, may be perceived from both, yet it is scarcely possible to distinguish the same part of them from each mountain. This will render the angles, consequently the distances inaccurate, and cause the observed points and heads in the draught, to extend more or less from the rest of the coast than they ought; and other distances and bearings that depend on them, will participate of their error. However, with a moderate degree of care, these errors will be inconsiderable, and the draught, in general, may agree sufficiently with observed latitudes and longitudes. Though by this method, an extensive survey may be carried on with more accuracy than by any of those before-mentioned, yet it has probably seldom been put in practice.

X. A Stasimetric Survey, is, when the mutual distances of *three*, or more, proper objects are carefully measured; and by means of these objects, the position and distance of all stations along the coast determined trigonometrically, each at its respective station alone, independent of one another.

By this manner of surveying, the errors which all the other methods are liable to, are avoided: for the distances along the coast, depending entirely on the distances of *three* objects chosen at pleasure, the most proper may be selected for that purpose: or, if proper objects are not to be had, others may be adapted to it

by signals, and their mutual distances previously determined to any degree of exactness. And if, by oversight, an error shall happen in any part, it is confined to that part alone, without affecting other parts; and may be corrected at any time, independent of the rest. This way of carrying on a geometrical survey, whether inland, or maritime, will appear evidently to be much more perfect than those *commonly* practised, and to surpass them, not only in accuracy, but in expedition; and it is the only method from which an exact extensive survey is to be expected. Other methods of surveying, are defective either in their foundation, or their procedure, or in both: and their errors, generally slip insensibly into all the parts of a survey, increasing as it goes on. Thence it is, that so few surveys have been continued beyond the extent of a large bay, or river; that the errors of former draughts are not to be corrected without an entire new survey; that some places have been surveyed again and again, yet in process of time are found to require it anew; and that so few draughts are to be met with, that answer the end for which they are intended, viz. to direct ships into safe anchorage without a pilot. The design of the following sheets is, to explain and recommend to more general practice, the *stasimetric* manner of surveying; to render future draughts of the sea-coast, more complete and serviceable than what have been hitherto offered to the public; and to direct such officers as have gone through a course of navigation, to make useful draughts of the bays or harbours they may occasionally visit, by means of such instruments *only*, as seamen usually possess.

USEFUL NOTES.

	Inches.		Feet.
The English foot is	12	A yard - - -	3
The Paris foot	12,788	A fathom - - -	6
The Amsterdam foot	11,172	A cable-length 100 fathom,	
The Danish great foot	12,465	or - - - - -	600
The Swedish foot	11,692		

	Yards.
An English statute mile is - - - - -	1760
A geometrical mile - - -	2038,6
A league, 3 geometrical miles - - -	6115,8
A French league, called Lieu legale - -	3996,46
A sea league of France - - - - -	6394,375
The Dutch league of 18000 Rhinland feet -	6186
The common German mile of 22800 Rhinland feet	7829,666
The league of Sweden 30000 Rhinland feet -	10310

Spanish league of	7572	Varas	17½ to a degree
Castellian do.	5000	do.	26½ do.
Portuguese do.	——	do.	19 do.

	English Miles.
A degree of the meridian in Britain -	69,5*
A degree at the equator - - -	67,75
A degree at the pole - - -	70,1
The chord of one degree of the meridian	69,150
The chord of half a degree - -	34,960
The diameter of the earth - -	7910, or, 41798117 feet†

The sun's semidiameter about the equinoxes is -	16′
About the middle of summer is - - - -	15′, 45″
About the middle of winter is - - - -	16′, 15″
The sun's greatest declination is - - -	23°, 28′, 9″

Geometrical Miles in a Degree of Longitude.

Deg. of Lat.	0	5	10	15	20	25	30	35	40
Miles & Parts.	60	59,77	59,09	57,95	56,28	54,38	51,96	49,15	45,96
Deg. of Lat.	45	50	55	60	65	70	75	80	85
Miles & Parts.	42,43	38,57	34,41	30,00	25,36	20,52	15,53	10,42	5,23

* In the Trigonometrical Survey of England, published in 1811, it is stated, that there are 60823 fathoms to a degree of the meridian in lat. 52° 34′ N. or 6082⅓ feet to a mile in this latitude.

† The French mathematicians made the mean diameter 41,828,814 feet, hence a mean degree is 365,528 feet, and a mile 6092 according to their admeasurements.

MARINE SURVEYING.

PART I.

GEOMETRICAL PRINCIPLES, AND OTHER PRE-REQUISITES OF SURVEYING.

MARINE SURVEYING.

CHAP. I.

Of the Geometrical Principles, and Pre-requisites of Surveying, in general.

PREVIOUS to undertaking a geometrical survey of the sea-coast, a proficiency in drawing will be found of advantage; at least so much of it, as to be capable of shading with India ink, or etching with a pen, *hills, rocks, cliffs, miniature buildings,* and the *coast-line* of a *map;* of sketching, from nature, the *out-lines* of them; and of taking distant views of the land, from sea. This can only be attained by practice, aided by a few instructions from a drawing-master. A surveyor who is not capable of sketching readily, and shading with some neatness, ought to have a draughtsman with him for that purpose.

He should also be acquainted with the elements of geometry and astronomy, at least with as much as is taught in a common course of navigation, to render him expert in protracting lines and angles, and calculating the cases of plain and spherical trigonometry by logarithmic tables, when the proportions are stated; and applying the former to the mensuration of distances, and the latter to that of spherical arcs and angles; of

taking the altitude of the sun, or a star, with a quadrant on sea and land, and from thence finding the latitude of a place; he ought also to know readily the principal fixt stars, especially such as are near the elevated pole.

He should be acquainted with the structure of the principal instruments used in surveying, such as the theodolite, the astronomical quadrant, and Hadley's octant, or sextant; and understand how to examine, adjust, and observe with them. If he is defective in any of the above particulars, his operations and observations, will be tedious and inaccurate, or his draughts void of neatness and elegance.

CHAP. II.

Trigonometrical Solutions.

THE cases in plain trigonometry that occur in surveying, are the four following.

1. Having two angles of a triangle, and a side opposite to one of them, to find another side; use the following proportion.

As the sine of the angle opposite to the side given,
Is to the sine of the angle opposite to the side required;
So is the side given,
To the side required.

2. Having two sides and an angle opposite to one of them, to find another angle; use the following proportion.

As the side opposite to the angle given,
Is to the side opposite to the angle required;
So is the sine of the angle given,
To the sine of the angle requir'd.

In the above cases, it will assist the memory to observe, that when a side is wanted, the proportion must begin with an angle; and when an angle is wanted, it must begin with a side.

3. Having two sides of a triangle, and the included angle, to find the other angles, and side; use the following proportion.

As the sum of the two given sides,
Is to their difference,
So is the tangent of half the sum of the two unknown angles,
To the tangent of half their difference.

Then half their difference (thus found) added to half their sum, will be the greatest of the two angles sought; which is the angle opposite to the greatest side. If the third side is wanted, it may be found by Solution 1. for two angles are known, and a side opposite to one of them.

4. The three sides of a triangle being given, from thence to find the angles; proceed by the following steps and proportions.

PLATE I. FIG. I.

1. Let A B C be the triangle: make the longest side A B the base: from C, the angle opposite to the base, let fall the perpendicular C D, on A B; which will divide the base into two segments A D and D B. Then,

2. Find the difference of A D and D B, by the following proportion,

As the base A B,
Is to the sum of the other sides (A C + B C)
So is the difference of the sides (A C − B C)
To the difference of the segments of the base (A D − D B.)

Half the difference of these segments (thus found) added to the half of A B, gives the greatest segment A D; or subtracted, leaves the least D B. Then,

3. In the triangle A D C, by Sol. 2. find the angle A C D: for the two sides A D and A C are known, and the right angle at D is opposite to one of them. The complement of A C D is the angle A. Then,

4. In the triangle A B C, you have two sides A B and B C, and the angle at A opposite to one of them; from thence (by Sol. 2.) find the angles C and B required.

CHAP. III.

Longimetrical Operations and Problems.

SEC. I. OPERATION I.

How to proceed in measuring a straight Line.

BEFORE any distance is measured trigonometrically, the length of *one straight line*, or distance, must be

found mechanically, by measuring it on a plane with a pole or chain. And as this mensuration is more or less exact, so will all the other distances be, that are deduced therefrom: consequently, the utmost care ought to be taken in measuring this *first distance*, or *fundamental base-line*, exactly; and all sources of error in the operation guarded against, as much as possible. The following directions will be found sufficiently exact for any survey.

1. Prepare a measuring-pole 20 or 30 feet long, precisely measured, square at one end, and a firm oblong ring at the other; or a chain 60 feet long: three poles, each 10 feet long, with white, or dark-coloured flags, to be tied to their tops, according as they may appear on the land, or between you and the sky: twelve poles, each 5 or 6 feet long, with a small flag on every fourth pole: four iron reels and stakes, such as gardeners use, with about 200 yards of small cord to each reel: twenty cylindrical pins for counting the number of poles, or chains, as you are measuring, each about 8 inches long, and sharpened at the point for piercing the ground easily: two wide-mouth'd canvas-pockets to carry the pins in, with strings for tying one of them round the middle of each of the measurers.

2. Pitch on a level plane, as long as can be had; such as a smooth sand, a salt-marsh, or (which is by far the best) a frozen lake.

In this plane, chuse such a direction for the line to be measured, as that the principal objects whose distances are required, may be seen from both its extremities; and that the greatest angle any of these objects make with it, may not be too oblique (or not above 140

degrees:) and, if convenient otherwise, let one end of this line run directly on some *remarkable part of a remote object;* so that, at the other end, its direction may be readily found at any time, by that object.

4. At each end of the line to be measured, set up a long pole, with a white flag flying at the top of that which appears on the land; or a dark flag on that which appears between you and the sky, or on white sand; so that they may be easily seen from each other: about the middle between these two, set up the other long pole, exactly in a line with them: and in a right line between these three, set up shorter poles all along, about 200 yards asunder, with a flag at the top of every fourth or fifth pole.

5. Stretch a line, or cord, from pole to pole, touching the same side of each pole as it passes them.

6. Immediately before you begin measuring along these lines, try the length of your measuring-pole, or chain, with a six-feet mahogany rod divided into feet, one of the feet into inches and tenths of an inch; the thickness of the wire of the ring at the end of the measuring-pole is not to be included in its length: count also the pins, put them in the canvas-pocket, and give them all to the leader, or foremost man, that carries the pole.

7. Lay the measuring-pole exactly along the cords, with the ring at the part you begin the measurement from, and its square-end foremost; at the square-end, just touching it in the middle, the leader is to set down a pin, and to go on with the pole until the follower (or

he who carries the ring-end) comes up to that pin, and puts the ring round it; then the leader laying the pole exactly along the cord, must thrust down another pin at the square-end. When that is done, he calls to the follower, who must take up the pin in the ring, put it into the canvas-pocket which is tied round his middle, and go on to the next pin, putting the ring round it; and when the leader has put down another pin, and called, then the follower must take his pin up, put it in the pocket, and go on to the next, as before; proceeding in this manner till all the pins are put down by one, and taken up by the other. Then leaving the measuring-pole exactly placed along the cord, without a pin at any of its ends, let the pins be counted over, to be sure that none are wanting, and the number marked down on paper, and all of them given back to the leader, to begin anew, by placing a pin at his end of the pole, as it lies along the line, which the follower is to put his ring round, and take up as before, and so go on till the whole distance proposed is measured.

The use of the ring at one end of the pole, instead of laying one pole at the end of another, is, to confine the measurers to more exactness; which will be found a necessary precaution for a surveyor.

A chain is not so exact for measuring with, when great precision is required, as a pole; but is much more conveniently carried; and, when care is taken to lay it straight, and to prevent the links from riding on one another, it will be found sufficiently exact on most occasions.

8. If the sand measured has a sensible and gradual declivity, as from high-water mark to low-water, then the length measured may be reduced to the horizontal

distance (which is the proper distance) by making the perpendicular rise of the tide, one side of a right-angled triangle, the distance measured along the sand the hypothenuse, and from thence finding the other side trigonometrically, or by protraction on paper; which will be the true length of the base-line. If the plane measured be on the dry land, and there is a sensible declivity there, the height of the descent must be taken by a spirit-level, or by a quadrant, and that made the perpendicular side of the triangle.

If, in a bay, one straight line of sufficient length cannot be measured, let two, or three lines, forming angles with each other, like the sides of a polygon, be measured on the sand along the circuit of the bay: these angles carefully taken with a theodolite, and exactly protracted, or calculated, will give the straight distance between the two farthest extremities of the first and last line. In taking the angles, let the center of the theodolite be placed exactly over the point where the two sides meet, by a plummet, or by dropping a stone down; and let poles be set straight up in the ends of the two sides that form the angle: for a small error in the angle, especially in the first, will often make a considerable error in the distance deduced from it.

Some surveyors measure their distances by a *wheel.* But as a wheel has a small serpentine motion from side to side, occasioned by the steps of the person that leads it; and besides, traces all the little inequalities of the ground, which, however smooth to appearance, no plane (if it is not smooth ice) is quite free from; this way of measuring cannot be reckoned sufficiently exact for a *fundamental base-line.* Much less is any distance measured by a line, or rope, to be esteemed sufficiently exact: for that will lengthen more, or less, by the strength ap-

plied in stretching it along the ground; and likewise lengthens, or shortens, by the various degrees of heat, or moisture in the air, or on the plane measured.

Operation II.

How to measure a straight Line on the Surface of the Sea.

1st. Prepare a measuring-line of strong cord, two or three hundred yards in length, with small pieces of cork of equal thickness made fast to it at small distances, all along, like a fishing-net, so that it may float straight on the surface of the water:* if the line has been well stretched, or much used before, it is the better: also prepare two ropes somewhat longer than the greatest depth of the water to be measured, with a pig of lead or iron ballast (which we shall call an anchor) 50 or 60 pounds weight, tied by the middle to one end of each rope, that when it is at the bottom it may be able to anchor a boat, and bear to be stretched straight without shifting the place of the anchor. Let the measuring-line be thoroughly wet immediately before you begin to use it, and then stretched on the water close by the shore, and its length measured there with a pole. Then, in the direction intended to be measured, take two remarkable sharp objects on the land in a line, one near the shore, the other as far up in the country as you can: if such are not to be had, place buoys on the water at proper distances in that direction.

* Cord of coir, or that made of other light species of hemp which floats naturally, would best answer this purpose.

2d. Take the objects, or buoys, in a line, and holding one end of the measuring-line fast on the shore, carry out the other in a boat, in that direction, till it is stretched straight at its full length by one man in the boat, and exactly at the end of the line let another man drop the anchor, which will mark one length of it. There keep the boat, and the end of the measuring-line, close to the anchor-rope, drawn right up and down, till another boat takes in the other end which was on the shore, and rows farther on, and lays it straight in the direction of the land-marks, or buoys, and there drops another anchor, which will mark the second length of the measuring-line. Go on thus till the whole proposed distance is measured; and immediately after let the measuring-line be again measured with a pole on the water near the shore, as at first, and if the lengths differ, take the mean between them for the true length. It is obvious, that to measure with any exactness this way, the sea must not only be smooth, but void of a swell, and of all stream of tide; either of which will hinder the line from lying straight. This method of measuring a straight line, may be convenient on some occasions; and if care is taken to keep the anchor-rope right up and down when the measuring-line is applied to it, will be found sufficiently exact for many purposes, but not for a *fundamental base-line*, from which other distances are to be deduced.

There is another way of measuring a straight line, mechanically, on the sea, which is so well known to seamen, that it is needless to describe it particularly here: and that is, by heaving the log over a ship's stern while she is under sail, and observing how many knots of the log-line runs out in half a minute; for the line is so divided that the ship will run (or, is supposed to run) so many miles in an hour, in a straight course; and twice

as much in two hours, and so on. But this conclusion is founded on three suppositions, neither of which are certain, viz. that the log remains in the same place during the whole half minute that the line is running out from the ship's stern; that the ship continues to sail with the same velocity, and also in the same direction, during an hour, or two, that she did during the half minute: the contrary of which, is more likely in most cases. For the log-line may shrink, or stretch, while it is running out; or may drag after the vessel by the weight of the line, or by not running easily and readily off the reel; the swell of the sea may alter the place of the log; and currents or streams of tide, stronger or weaker below the surface than at it, an unsteady helm, lee-way, and varying winds may change the direction, or celerity of the ship's motion; for neither of which can any *certain* allowance be made. This way, therefore, of measuring a straight line, or distance, is not to be depended on as exact: but is mentioned here, because rocks, shoals, or islands sometimes lie so far from the coast, that there is no other way of forming any notion of their distance. If any such distance is to be measured after this manner, let the log-line be thoroughly wet when it is measured; let the length between each knot be 51 feet, which is the hundred-and-twentieth part of a geometrical mile, as half a minute is $\frac{1}{120}$th part of an hour; chuse neap-tide, as much slack water as can be got, and a moderate breeze of following wind; let the line be run off the reel so as never to be stretched quite straight; and if the half-minute is measured by a watch that shews seconds, rather than by a glass, it will generally be more exact. Perhaps one second should be allowed for the loss of time in calling out at the beginning, and stopping it at the end of the time; except the person who holds the

watch can observe the going out of the red rag at the beginning, and also stop the line himself at the end of the time; which does not seem difficult.

OPERATION III.

To find the Distance of two Places, by the Flash and Report of a Gun.

Stand in one of the places with a stop-watch in your hand; the watch stopt with the second-hand at 60, and your finger at the stopper: let a gun be fired at the other place, at an appointed time. When you see the flash, or smoke, that instant set the watch a-going; and immediately as you hear the report, stop it; and it will shew the number of seconds of time between the flash and report. By that number multiply 1142, and the product will be the distance of the two places in feet: for sound is found to move 1142 feet in a second.

This operation should be repeated two or three times, for more certainty; and the exactest of them, if it is certainly known, fixed on for the true time: or a mean of all, if no distinction can be made. It should also be performed when there is little wind; for a breeze of wind, with or against the sound, is found to quicken or retard it a little. It is likely that some small variation may also arise from different degrees of density, and elasticity in the air; the former retarding, and the latter accelerating sound. But this has not yet been ascertained by experiments.*

* Mr. Millington's experiments make the velocity of sound to be nearest 1130 feet in a second, accelerated or retarded *a little* by the direction of the wind: but the state of the barometer made no dif-

By this way of measuring distances, such as are not more than two miles cannot be reckoned exact; because of the difficulty of setting the watch a-going, and stopping it at the precise times;† which may cause an error of, *at least*, one quarter of a second or 90 yards, which is too much in so short a distance. If the distance is four miles, or more, it may be reckoned sufficiently exact for ordinary purposes. I incline to think, if half a second is allowed for the time lost in stopping the watch and setting it a-going, that, after a little practice, one may measure two miles pretty exactly by this method; provided the state of the air is such as it was when the experiments for determining the velocity of sound were made.

ference in its velocity: Count La Place says, that the velocity of sound in the atmosphere is increased by heat; that it moves swifter in dense, than in rarefied media, its velocity being about $4\frac{1}{2}$ times swifter in rain water than in air, and swifter in salt water than in fresh water, and much swifter in brass than in water.

It is to be wished that farther experiments were made by some careful person, who would attend to the several circumstances of the air and wind that might affect the celerity of sound, and publish the effect of each, for the conveniency of surveyors, and the farther satisfaction of the curious.

† A chronometer which beats *half seconds* is more convenient than a watch, as an observer (without an assistant) may place the chronometer sufficiently near to *hear* the beat of the half-seconds, and consequently, can easily count the number of them interlapsed from the time of seeing the flash to that of hearing the report of the gun.

Corr. f. [illegible]

SECTION II.

Longimetrical Problems.

PROB. I.

Having the Length of a Base-line X Y, *to find the Distance of two, or more remote Objects* A, B *and* C, *from it, and from each other.*

PLATE I. FIG. 2.

BY PROTRACTION.

With a theodolite, or Hadley's quadrant, at one end of the base-line X, take the angles Y X A, Y X B, Y, X C; at the other end Y, take the angles X Y A, X Y B, X Y C, and write them down in a book. Then draw a line on the paper of sufficient length, and in a convenient position: pitch on any convenient part of that line for the point X; and taking the length of X Y from a scale of equal parts, set it off from X to Y: at X draw the observed angle Y X A; and at the other end Y, draw the observed angle X Y A, and the intersection of these two lines will be the point A; and X A, measured on the scale from which X Y was taken, will be the distance of A from X. In the same manner the points B, or C, may be determined; and the length of A B, or B C, measured on the scale, will give the distance of A from B, or B from C: and so for any other points whose distances are required, whether they lie on one side of the base-line or the other.

Let it be observed, that if X Y is drawn on the paper

according to its magnetic direction, then the points A, B, and C, will likewise be in their magnetic directions from X and Y, and from each other.

To render protracted distances more exact, the length of the base-line should not be less than one-seventh, or one-eighth part of the distance of the farthest object; none of the angles should be too oblique, or above 140 degrees; the protractor should be at least 12 inches diameter, and each degree subdivided into four parts; or more minutely by an index with a vernier scale on it.

By Calculation.

In the triangle X Y A there are given the measured side X Y, and the observed angles Y X A and X Y A, and therefore the angle Y A X, their compliment to 180°; that is, two angles and a side opposite to one of them: therefore, (by Trigon. Sol. 1.) the side X A may be found. In the same manner, in the triangle X Y B, may X B or Y B be found. Then, in the triangle A X B, there are given the two sides X A and X B, and the included angle (the difference of the observed angles Y X A and Y X B;) hence A B may be found, by Trigon. Sol. 3.

PROB. II.

To find the Distance of two Places, when the Latitudes, their Bearing by the magnetic Needle, and the Variation of the Needle are known.

Plate I. Fig. 3.

By Protraction.

1. Draw the Line X A right up and down on the paper, for the magnetic meridian.

2. Pitch on a convenient point X in that line, to represent one of the places; and at the point X make the angle A X B equal to the variation, and on the east side of X A, if the variation is west; but on the west side of it, if the variation is east; and X B will be the true meridian.

3. At X make the angle A X Y equal to the magnetic bearing of the other place from X.

4. Take from a scale of equal parts the difference of latitude of the two places in geometrical miles, and set it off on the true meridian from X to B.

5. At B raise a perpendicular B Y cutting X Y in Y; and the point Y will be the other place; and X Y, measured on the scale, will be the distance of the two places in geometrical miles.

If their distance is wanted in English miles, say, as 60 is to 69,5 so is their distance in geometrical miles, to their distance in English miles: for 69 and a half English miles make 60 geometrical miles.

By Calculation.

In the right-angled triangle Y B X, there is given the angle Y X B, in this case, equal to the bearing of Y and variation of the needle together; and therefore the angle at Y is also known; and the side X B opposite to one of them: hence (by Trigon. Sol. 1.) X Y is found.

In this problem, the greater the difference of latitude of the two places is, and the nearer they are to the same meridian, their distance will come out the more exact.

For, an error of half a minute can hardly be avoided in the latitudes, though taken with the best portable quadrant; nor an error of 15 minutes, sometimes, in the bearing of Y from X, though taken with the best needle: more especially, as the polarity of needles is found to vary in the same place above 20 minutes in a day, according to the state of the atmosphere, as is thought. To be sufficiently exact, therefore, the difference of latitude of the places should not be less than 15 miles, and the angle B X Y no greater than 45 degrees.

The accuracy of this problem depends chiefly on the accuracy of the two latitudes: for supposing the needle to be twenty minutes wrong, it will not occasion an error of a quarter of a mile in a distance of thirty miles, if the intersection of the bearings is not oblique: and therefore, if the quadrant with which the latitudes are found, and the observations likewise, are not known to be good, the distance deduced from them is not to be depended on as exact.

Here it may be observed, that the distance between X and Y, thus found, is the length of an arc of a great circle of the earth between these two stations, and not the straight line, or shortest distance. The chord of that arc is the true distance to be used in surveying. Therefore, instead of X Y measured by two latitudes, take its chord, which is twice the sine of half the arc. The proportions between the following arcs and their chords were found by the tables of natural sines by saying, as the radius of the tables, is to the natural sine of half the given arc, so is the radius of the earth to a fourth number; which doubled, is the chord required. If X Y therefore is found by two latitudes to be 69½ English miles, the chord of that arc, or shortest distance between X and Y is only 69 miles and 150 yards:

if the arc is $34\frac{3}{4}$ miles, its chord is $34\frac{1}{2}$ miles and 80 yards: if the arc is 20 miles, its chord will be $19\frac{3}{4}$ miles and 264 yards nearly. If the arc measured is less than 20 miles, its chord may be reckoned the same with the arc.

If, in practice, it shall happen that Y cannot be seen from X; or that, because of their distance, both points cannot be included in the same draught; in these cases, to avoid the trouble and loss of time in finding the latitude of another place instead of X, pitch on some other station nearer Y, as S, whose distance and direction from X is determined on the draught, and where Y can be seen: find its latitude, by letting fall the perpendicular S b on the true meridian drawn through X; and b X measured on the scale and reduced to geometrical miles, will be the difference of latitude between X and S: which added to, or subtracted from the latitude of X (as S is northward or southward of it) will be the latitude of S. Then, having the latitude of S and of Y, find their distance, as before directed.

PROB. III.

Having the Distances of three Points, A, B *and* C, *either in a straight Line, or forming a Triangle; at any Station* S, *without the Triangle, to find its Distance from these Points.*

PLATE I. FIG. 4.

BY PROTRACTION.

At S, with a theodolite, or Hadley's Sextant, take the angles A S C and C S B: then, making A B (the

side which subtends the greatest angle, and is opposite to the middlemost point) the base; at B, make the angle A B D equal to the angle A S C, and lying on that side of A B which is farthest from S; and at A, make the angle B A D equal to C S B, intersecting A D in D; draw a circle passing through the three points A, D, B, and it will also pass through S: draw out the line C D, and it will cut the circle in S, the point of station required.

Demonstration. The angles A S D and A B D are equal, for they stand in the same segment A S B D (Eucl. b. III. prop. 21.) A S D is therefore equal to the first observed angle. In the same manner, B S D and B A D are equal, for they stand in the same segment D A S B; and D S B is therefore equal to the other observed angle; and S the point of station, and S A, S C, S B, the distances required.

By Calculation.

1. In the triangle A B C, the sides being known, the angles are found by Trig. Sol. 4.

2. In the triangle A B D, the side A B is given by supposition; and the angles D A B and D B A equal, respectively, to the corresponding observed angles; therefore the angle at D is also known: from thence (by Trig. Sol. 1.) find the side A D. Then,

3. In the triangle C A D, the sides C A and A D are known, and the included angle C A D (which is what

B A D wants of B A C) from thence (by Trig. Sol. 3.) find A C D. Then,

4. In the triangle A S C, two angles A S C and A C S, and the side A C opposite to one of them, are known; from thence (by Trig. Sol. 1.) A S may be found. In the same manner may S B or S C be found.

If the point C is on the other side of A B, towards S, as at k, or in the line A B, it is obvious that S may be found in the same manner; the same protraction, and calculation, serving for each of these cases.

Fig. 5.

If it shall happen that the points C and D fall so near each other, that in protracting, C D is produced with uncertainty, which will happen when the point C lies beyond A B; and the observed angle A S C is nearly equal to the angle A B C, and the other observed angle C S B is nearly equal to the angle C A B. In which case take the point C on the side of A B which is next to S, as at k: or, shift your station nearer to, or farther from C, and find the position of that station, and then its distance from S. Or, on C B, make the angle C B D equal to the observed angle A S C, and the angle B C D equal to the supplement to 180° of both the observed angles A S B; then the intersection of these sides will be the point D: draw a circle passing through the points B, C, D,; and A D produced, will cut that circle in S, the point of station required. For,

B S D C being a quadrilateral figure inscribed in a

circle, B S D is the supplement of B C D to 180° (by Eucl. b. III. prop. 22.) and by construction equal to the two observed angles: and A S C and D B C being equal, as standing on the same arc D C; A S C must be the first observed angle, and C S B (the remainder of A S B) the other; and S therefore the point of station.

Here it may be observed, that the distance of an object found in this manner by Hadley's sextant, is the real aerial distance in a straight line from the eye to the object; but it is the level distance that is necessary in surveying. If the object therefore is on a hill, or eminence, that is near, and of considerable height, it will be greater than is required in surveying, as the hypothenuse of a right-angled triangle is greater than the base: To find the level distance, or distance of a point within the base of the hill, right below the object on the top of it; pitch on some remarkable thing that lies below, and in a line with the object, and take the angle with that instead of the object on the hill.

PROB. IV.

Having the Distances of three Points A, B *and* C, *that form a Triangle, at any Station* S, *in the Direction of one of the Sides, to find its Distance from these Points.*

FIG. 6.

BY PROTRACTION.

Take the angle A S B with an instrument; subtract that angle from C A B, and the remainder will be the

angle A B S; for C A B is the external angle of the triangle A B S, and therefore (by Eucl. b. I. prop. 32.) is equal to A S B and A B S taken together. Then, at B, on the side B A, draw the angle A B S; produce C A, and it will intersect B S in S, the point of station; and S A, S C, S B will be the distances required.

By Calculation.

In the triangle A B S, the angle at S is found by observation; the angle B A S is the supplement of C A B to 180°, and therefore the angle A B S is also known: that is, in the triangle A B S, all the angles and a side are known; from thence the other two sides S A and S B are found by Trig. Sol. 1.

PROB. V.

Having the Distances of three Points A, B *and* C, *forming a Triangle, at any Station* S, *within the Triangle, to find its Distance from these Points.*

Fig. 7.

By Protraction.

Make any one of the sides, suppose A B, the base: take with an instrument the angles A S C and C S B, which the two other sides subtend: at B make the angle A B D (lying without the base of the triangle) equal to the supplement of A S C to 180°; and the angle B A D equal to the supplement of C S B: these sides produced,

will intersect in D. Draw a circle passing through the three points A, D, and B; join C D, and it will intersect the circle in S, the point of station; and S A, S C, S B will be the distances required.

Demonstration. The angles A S D and A B D are equal, for they stand in the same segment A S B D (Eucl. b. III. prop. 21.) therefore A S C, the supplement of A S D, is also the supplement of A B D; and, by construction equal to the first observed angle. Again, B S D and B A D are equal, for they stand in the same segment B S A D; and B S C, the supplement of B S D, is also the supplement of B A D; and therefore, by construction, equal to the other observed angle; and S the point of station; and S A, S B, S C, the distances required.

By Calculation.

1. In the triangle A B D, the angles B A D and A B D are known, being respectively supplements of the observed angles A S C and B S C; and therefore their supplement A D B is known; and the side A B opposite to one of them: from thence (by Trig. Sol. 1.) find the side A D. Then,

2. In the triangle C A D, the sides A C and A D are known, and the included angle C A D (equal to C A B and B A D together) from thence find the angle A C D, by Trig. Sol. 3.

3. In the triangle A C S, the angles at C and S are

known, and the side A C, opposite to one of them; from thence (by Trig. Sol. 1.) find A S and C S.

PLATE I. FIG. 8.

If the point of station is in any of the sides, as at S in the side A B; take the angle which one of the other sides, suppose A C, subtends there; and on any part of A B, as D, make the angle A D E equal to the observed angle A S C: through C, draw the line C S parallel to D E cutting A B in S, and S will be the point of station. For C S and D E being parallel, the angles A S C and A D E are equal (Eucl. b. I. prop. 29.) A S C is therefore the observed angle, and S the point of station.

On account of the great utility of these last problems in surveying, I shall shew a general, and more ready method, of finding, by protraction, the point of station S, in the three principal cases foregoing; communicated by Capt. *John Campbel** of his Majesty's navy.

1. *When the observed angle* A S B *fig.* 9, *and* 10, *is less than* 90 *degrees.* From the points A and B draw the angles B A m and A B M, lying towards S, each equal to the complement of the observed angle A S B, and the intersection of the lines A m, B M, will be the center m of the circle A S B. From the extremities of either of the other sides of the triangle A B C, as B C, draw the angles B C n and C B N, each equal to the complement of the observed angle C S B, and the lines C n, B N, will intersect each other in n, the center of

* Afterward Admiral Campbel, an officer of great science.

the circle C S B; and the point of intersection of these two circles shall be the required station S. For,

The angle A S B being less than a right angle, the line A B will divide the circle A M S B into two unequal segments, in the greatest of which (A M S B) S must stand (Eucl. b. III. prop. 31.) and therefore the center, m, which is always in the greatest segment, will be on the same side of A B that S is. Also, in the triangle m A B, the angles A B m and B A M together, are the supplement of the angle A m B to two right angles; but A B m and B A m are (by construction) the supplement of twice A S B to two right angles; A m B is therefore double the angle A S B; and standing on the same arc (by Eucl. b. III. prop. 30.) S must be in the circumference of the circle A S B.

In the same manner it may be shewn that S is in the circumference of the circle C S B, and therefore it must be in their intersection.

2. *When the observed angle* A S B (Fig. 11.) *is more than* 90 *degrees:* then subtract 90 from it, and make the angles at A and B each equal to the remainder; and to lie on the side of A B, which is farthest from S; and m, the point of intersection of these two lines will be the center of a circle, which will pass through A, B and S. For, the angle A S B being more than 90 degrees, the line A B will divide the circle into two unequal segments, in the least of which (A S B) the point of station S must be; and the center m, therefore, on the side of A B opposite to S. Also, the angle m A B = m B A = (by construction) to the difference between A S B and a right angle: also the angle n C B = n B C = $\overline{\text{CSB} - 90^\circ}$.

PROB. VI.

Given the sides and angles of the Triangle A B C, *to find the distance of any Station* S, *outside of the Triangle, when the angle* A S B *is more than* 90°.

PLATE I. FIG. 12.

Subtract 90° from angle A S B, and on that side of the triangle A B C farthest from S, draw the triangle A m C, making the angles C A m and A C m, each equal to the remainder: the point m will be the centre of a circle which will pass through A S B.

From the points B and C draw angles B C n, C B n, each equal to the complement of angle B S C, and n will be the centre of a circle passing through B S C. The point of intersection s, of these two circles, will be the station required.*

But the surveyor must be careful, that the three objects forming the triangle A B C, which he intends to make use of to find station S, do not come on, or near, the circumference of the same circle that passes through it and these three objects; as in such case, it will be impossible for him to determine station S. For,

Let A B C, fig. 13, be three fixed objects; describe a circle to pass through them; now, at whatever part of that circle an observer is situated, the relative angles to

* This problem being of great utility in nautical surveying, (but omitted in the original work of Mr. M'Kenzie) is now supplied and inserted at the recommendation of my friend Mr. J. Walker, of the Hydrographical Office, Admiralty.

those objects will be the same, (Eucl. b. III. prop. 21.) as S, s. Or,

Should a circle cutting two of these objects, pass near the third, as A B C, fig. 14, the two circles will nearly coincide, and intersect each other in an angle as at S, s, too acute to determine the point of station.

The Point of Station S *found instrumentally.*

The point S may be readily laid down on a draught, by drawing on a loose transparent paper indefinite right lines S A, S B, S C, at angles equal to those observed; which being placed on the draught so as each line may pass over, or coincide with, its respective object, the angular point S will then coincide with the place of observation. Or,

Provide a graduated semicircle of brass, about 6 inches in diameter, having three radii with chamfered edges, each about 20 inches long, (or as long as it may be judged the distance of the stations from the three given objects may require) one of which radii to be a continuation of the diameter that passes through the beginning of the degrees on the semicircle, but immoveably fixed to it, the other two moveable round the center, so as to be set and screwed fast to the semicircle at any angle. In the center let there be a small socket, or hole, to admit a pin for marking the central point on the draught. When the sloped edges of the two moveable radii are set and screwed fast to the semicircle, at the respective degrees and minutes of the two observed angles, and the whole instrument moved on the draught until the edges of the three radii are made to lie along the three stasi-

metric points, each touching its respective point, the center of the semicircle will then be in the point of station S; which may be marked on the draught, through the socket, with a pin. Such an instrument as this may be called a *station pointer*,* which would be convenient for finding the point of station readily and accurately, except when the given objects were near, when the breadth of the arch, of the radii, and of the brass about the center of the semicircle might hinder the points from being seen, or the radii to be placed so as to comprehend a very small angle between them.

PROB. VII.

Having the Distance and Position of two Points A *and* B; *at two Stations,* S *and* s, *to find the Distance of these Stations from each other, and from the given Points.*

PLATE I. FIG. 15.

1. At S, take the angles A S s and B S s; and at s, take the angles A s S and B s S. Then,

2. Making S s any length, suppose 10, take it from a scale of equal parts, and draw it on paper: and at S and s, protract the respective observed angles, and the points A and B will be determined in proportion to the assumed length of S s. Then, taking the protracted length of A B from the scale, say,

* Station pointers are now made by most of the mathematical instrument makers, and frequently used by surveyors.

As the protracted length of A B,
Is to the assumed length of S s,
So is the true length of A B,
To the true distance of S *and* s.

3. The distance S s being thus found, and the angles at S and s observed, the distances S A, S B and s A, s B, may be taken from the proper scale, after a new protraction; or calculated, as in Prob. I.

PROB. VIII.

Having the Distance and magnetic Bearing of two Points, A *and* B *protracted; at any Station* S, *(not very oblique to* A B) *to find its distance from these Points, by the Needle.*

PLATE I. FIG. 16.

At S, with a good magnetic needle, take the bearings of A and B, in degrees and parts of a degree: then, from these points draw out their respective bearings in the opposite direction, towards S: that is, if A bears exactly north, draw a line from the point A exactly south; if it bears east 10, or 20 degrees southward, draw the line west 10, or 20 degrees northward, and so for any other bearing. Draw the opposite bearing of B in the same manner; and S, the intersection of these two lines, will be the point of station; and S A, S B, the distances required.

This is an easy and ready way of finding the distance of any station, from two places whose distance has been accurately determined before; and will be found con-

venient very often in the course of a survey; and sufficiently exact on most occasions; provided the places, A and B, are not very remote from the station, nor the intersection of their bearings too oblique. If the needle is good, a distance of 20 miles is not to be reckoned too far, when the angle subtended by the two places is not less than 50 degrees, nor more than 140.

CHAP. IV.

On the Examination of Surveying Instruments.

THE accuracy of geometrical mensuration depends so much on good instruments, that it is a very necessary qualification in a surveyor, to be capable of examining and adjusting them. Some instruments are made in such a manner, as to render it very troublesome to find whether all their parts are exact or not; surveyors therefore, are too apt to take them on trust, without examination: but this often creates more trouble in their business, and occasions errors which they can neither account for, nor correct. If surveyors were more cautious, instrument-makers would become more careful.

How to examine the Accuracy of a Theodolite.

PLATE I. FIG. 17.

1. On a level about a mile or two long, such as the margin of a lake, or the high-water mark along a well-

sheltered sand, when the sea has been smooth a little before, set three upright poles* A, B, C, exactly in a line; and about half a mile, or a mile from each other. If it can be done, let the line run exactly on some remarkable part of a distant object.

2. Set the theodolite horizontal, with its center right over the hole B, where the middle pole stood: set the index at 0 degrees, and turn the theodolite till, through the telescope, you see the pole A at the vertical wire; then screw the instrument fast in that position. Next turn the index to 90, and remark some distant object exactly at the vertical wire of the telescope: if no such object is to be seen, set up a pole D in that direction half a mile off, or as far as it can be seen.

3. Then set the index at 0 deg. again, and turn the instrument till you see the pole C, through the telescope exactly at the vertical wire: screw the instrument fast in that position; turn the index to 90, look through the telescope, and if the object, or pole, D, is exactly at the vertical wire, then is the index right centered, and the graduated circle exactly divided into four quadrants; for the line D B, standing on the right line A C, makes the angles on each side, A B D and C B D, equal. But, if the object is not seen exactly at the vertical wire, observe how much it is over, or falls short of it, and half that difference is the error of the division at 90.†

* To place a pole upright; stand straight up with your feet close together; put one end of the pole between your toes, and take the other up along your breast and chin, and in that posture press it down with both hands till it stands firm.

† By a proper application of the above principles, the graduation of instruments, large or small, may be rendered as accurate as the

4. To find if the graduations of each quadrant are right: apply the beginning of the divisions of the vernier scale, to the beginning of each degree of the theodolite; and if the end of the divisions of the vernier reaches the respective degree precisely in each application, then the graduations of the theodolite are right; otherwise one, or more, are wrong. If each degree is subdivided, these subdivisions may be examined in the same manner. This is a very easy and nice way of examining the graduations of any instrument that has a vernier scale on it.

To find if the wire of the telescope is exactly vertical: hang up a plumb-line at some distance; set the theodolite level; look through the telescope to the plumb-line when at rest, and if the upright wire moves exactly along the plumb-line when the telescope is elevated or depressed, then is the wire exactly vertical. If the wire crosses the plumb-line in any part, turn it by the proper screws, or pins, till you get it exact.

Cautions in taking Angles with a Theodolite.

1. Spread the legs that support it pretty wide, and thrust them firm in the ground, especially on sand, or soft ground; that they may not yield, or sink unequally during the observations.

2. Set it level; otherwise the angles will not be precise.

materials made use of will admit, and the least degree of inaccuracy in them readily discoverable; as could be easily shewn, were this a place for it.

3. Screw the ball firm in the socket, that in turning the index, the diameter of the theodolite may not vary from the object it was directed to; which, without care in the observer, or in the structure of the instrument, will happen frequently.

4. The angles should be taken twice over, and read off by different persons.

5. It would often prevent mistakes in angles, and contribute to despatch, if a theodolite was provided with two telescopes, one above its plane, and the other below it; the lower one to move round only when the circumference of the theodolite was turned; by which means it would be readily seen through this telescope when the diameter shifted from the direction of the object it was fixed at. Also, if these telescopes shewed objects erect, and in their natural position, instead of being reversed and inverted; and if the telescopes magnified more than they commonly do.

Of Hadley's *Sextant.*

This instrument may be used with great advantage in marine surveys, on most occasions; being more portable, more readily applied to the taking of angles, and generally more accurately and minutely divided than theodolites: an observer is less liable to mistakes with it; and, which is a very material advantage, he can take angles with it at sea, as well as on land. There are two inconveniences attending the use of it; one is, that small, or obscure objects, especially if the ground rises beyond them, cannot be easily perceived in the glasses: the

other is, that no angle above 120 degrees can be taken by it at once, and that sometimes there are not proper intervening objects for dividing a larger angle into two.*

A brass sextant of eight inches radius, will be found as convenient a size as any for a surveyor. It should be provided with a small telescope, that can be taken off and put on at pleasure, for observing celestial bodies; and with a tube for terrestrial objects, which is better than a thin plate of brass with holes in it. There should likewise be a screw at the end of the index, to bring the two objects exactly to one another, after they have been first brought near by the hand; and if a staff with a ball and socket is fitted to it, and a belt to support the whole, it will be found convenient to the observer. The telescope and tube should be so contrived, as to be raised occasionally as high as the middle of the transparent part of the horizon-glass, for observing objects that are bright enough to be seen by reflection from it.

Each of the glasses belonging to the sextant should have its two surfaces perfect planes, and parallel to one another; and the index and horizon-glasses should stand perpendicular to the plane of the instrument. These particulars of the glasses ought to be carefully attended to, and adjusted by the instrument-maker, before he gives the sextant out of his hand: for though defects in the glasses may be detected by proper examination, yet the want of parallelism and plainness cannot be corrected without new glasses; and, to adjust the inclination, is troublesome to young practitioners. The first thing,

* Circles, or quintants, are now often used to measure large angles in surveying.

however, which a beginner should learn is, the easiest way of holding the sextant for taking observations, and of bringing the reflected image and object near each other by moving the index: although this requires practice more than precept, the following directions may be found useful to such as have not experience.

How to hold the Sextant for taking Angles.

Whether the sextant is to be held vertically, or horizontally with its face uppermost, it should be supported chiefly with the right hand, and the index governed by the left, in the following manner.

For horizontal angles; hold the instrument horizontally, with the face uppermost, and place the thumb of the right hand against the edge of the curved part on which the sight-vane stands, but so as not to touch the index when it is set at 0, or the beginning of the degrees; and extend your fingers across the back of the instrument, so as to lay hold of the opposite edge of the additional part on which the horizon-glass stands, placing the fore-finger on one side, the two next fingers on the other side of the greatest swelling, and the point of the little finger against the back of the instrument. When there is no staff and belt, horizontal angles are easiest taken sitting with the right ancle on the left knee, and the right elbow supported by the right knee. The index may be managed by placing the thumb of the left hand against the outer edge of the graduated arc, and keeping the end of the index betwixt the two first fingers: or by taking the end of the index between the fore-finger and thumb, and resting the other fingers against the

middle bar, or against the far side of the instrument, as a very little practice will direct.

For taking vertical angles: hold the sextant in a vertical position, with the thumb and fingers of the right hand, as before, for horizontal angles; or as the form of the swelling at the horizon-vane enables you to take the surest hold; rest the extremity of the arc against your breast, and that will enable you to manage the index more easily with your left hand.

When angles are to be taken with the instrument inverted, or with the face below, then it must be held and supported chiefly by the left hand, and the right reserved for governing the index, in the following manner. Hold the sextant horizontal, with its face downward: place the thumb of your left hand on the center-plate at the back of the index-glass; the three first fingers above the center-part, and the inside of the little finger against the edge of it; and when the eye is applied to the sight-vane, press the edge of that side against your forehead: take care that the little finger, by being too low, does not intercept the sight of any of the objects.*

In all observations, the sextant should be held in a plane passing through the eye of the observer and the two objects whose angular distance is to be found.

Rules for examining the Glasses of a Sextant.

To find whether the two surfaces of any one of the reflecting glasses are parallel: apply your eye at one end of it, and observe the image of some distant object

* Sextants are *now* much improved, and generally have a proper handle, which on most occasions is grasped or held by the left hand, or as found most convenient.

reflected very obliquely from it; if that image appear single and well defined about the edges, it is a proof that the surfaces are parallel: on the contrary, if the edge of the reflected image appears misted, as if it threw a shadow from it, or separated like two edges, it is a proof that the two surfaces of the glass are inclined to each other. If the image in the speculum, particularly the sun, is viewed through a small telescope, the examination will be more perfect.

To find whether the surface of a reflecting glass is a perfect plane. Chuse two distant objects as nearly on a level as can be had: hold the sextant horizontal, with your eye at the sight-vane, view the left hand object directly through the transparent part of the horizon-glass, and move the index till the reflected image of the other is seen below it in the silvered part; make the two images unite just at the line of separation: then turn the sextant round slowly in its own plane, so as to make the united images move along the line of separation of the horizon-glass: if the images continue united, without receding from each other, or varying their respective position, the reflecting surface is then sufficiently plane.

To find if the reflecting glasses* are perpendicular to the plane of the sextant. Move the index near the middle of the arc of the sextant; apply your eye obliquely near one end of the reflecting glass, so as to see part of the arc of the sextant by reflection, and part of it by direct vision: if the two parts of the arc form one uniform curve, without appearing in the least above or below each other, then is the glass perpendicular to the

* Or the great speculum more particularly.

plane of the sextant. If the image of the arc seen by reflection is higher than the arc seen direct, then the glass inclines forward; in which case, slacken the fore-adjusting screw of the glass, and screw in the back one. If that part of the arc seen direct is higher than its image seen by reflection, then the glass leans backward, and the back-adjusting screw must be slackened, and the fore one screwed in.* Be careful, in all cases where one screw draws, or pushes against another, to slacken one of them before you screw up the other.

To find if the two surfaces of a red or darkening glass, are parallel and perfectly plane. This must be done by means of the sun when it is near the meridian, in the following manner. Hold the sextant vertically, and direct the sight to some object in the horizon, or between you and the sky, under the sun; turn down the red glass, and move the index till the reflected image of the sun is in contact with the object seen direct: fix then the index, and turn the red glass round in its square frame; view the sun's image and object immediately, and if the sun's image is neither raised nor depressed, but continues in contact with the object below, as before, then the surfaces of the darkening glass are true.

How to observe the horizontal Angle, or angular Distance, between two Objects.

First adjust the sextant, and if the objects are not small, pitch on a sharp top, or corner, or some small

* Some of the great speculums on sextants, are permanently fixed without adjusting screws.

distinct part in each to observe; then having set the index to 0 deg. hold the sextant horizontally, as above directed, and as nearly in a plane passing through the two objects as you can; direct the sight through the tube to the left hand object, till it is seen directly through the transparent part of the horizon-glass: keeping that object still in sight there, move the index till the other object is seen by reflection in the silvered part of the horizon-glass; then bring both objects together by the index, and by the inclination of the plane of the sextant when necessary, till they unite as one, or appear to join in one vertical line, in the middle of the line which divides the transparent and reflecting parts of the horizon-glass: the two objects thus coinciding, or one appearing directly below the other, the index then shews on the limb the angle which the two objects subtend at the eye of the observer. This angle is always double the inclination of the planes of the two reflecting glasses to one another; and therefore every degree and minute the index is *actually* moved from 0, to bring the two objects together, the angle subtended by them at the eye will be twice that number of degrees and minutes, and is accordingly numbered so on the arc of the sextant; which is really an arc of 60 degrees only, but graduated into 120 degrees.

The angle found in this manner between two objects that are *near* the observer, is not precise; and may be reckoned exact only when the objects are above half a mile off. For, to get the angle truly exact, the objects should be viewed from the center of the index-glass, and not where the sight-vane is placed; therefore, except the objects are so remote that the distance between the index-glass and sight-vane vanishes, or is as nothing

compared to it, the angle will not be quite exact. This inaccuracy in the angle between near objects is called the *parallax* of the instrument; and is the angle which the distance between the index-glass and sight-vane subtends at any near object. It is so small, that a surveyor will seldom have occasion to regard it: but if great accuracy is required, let him chuse a distant object exactly in a line with each of the near ones, and take the angles between *them*, and that will be the true angle between the near objects. Or, observe the angle between near objects, when the sextant has been first properly adjusted by a distant object; then adjust it by the left hand object, which will bring the index on the arc of excess beyond 0 degrees: add that excess to the angle found between the objects, and the sum will be the true angle between them. If one of the objects is near, and the other distant, and no remote object to be found in a line with the near one; adjust the sextant to the near object, and then take the angle between them, and the error of parallax will be removed.

To observe the Sun's Altitude on the Sea, or on Shore at the Edge of the Sea.

Adjust the sextant by the horizon (as is afterwards directed:) raise the telescope, or tube, as high as the middle of the transparent part of the horizon-glass; set the index to 0 degrees; turn in the darkening-glasses, or only one of them if the sun is not very bright; turn your face directly to the sun; hold the sextant in a vertical position, with its plane passing as near through the sun's centre and the nearest part of the horizon as you can, and direct the sight to that part of the horizon;

then move the index from 0 on the limb, till you see the image of the sun come down toward the horizon. If there is any difficulty in finding that image, it is either because the sun is obscured by a cloud, or because the sextant is not in a vertical plane that passes through the sun, which must be discovered by glancing at the sun with your naked eye, and in the latter case, by turning your body more direct to the sun. Having brought the sun's image almost down to the horizon, then, with your left hand, swing the sextant backward and forward, from one hand to the other, and the image of the sun will seem to describe the arc of a circle convex to the horizon: move the index till the lower edge of the image just touches the horizon, when it is at the lowest point of the arc apparently described, then stop the index, and mark down the degrees, minutes, and seconds, shewn on the limb, which call the *observed altitude.* When this is corrected by the sun's semi-diameter, the dip of the horizon, the refraction, and the index-error, if there is any, it will give the true altitude of the sun's centre.

How to take an horizontal Angle between two Objects with the Sextant inverted.

To observe in this manner: set the index at 0, and hold the instrument as above directed; direct the sight to the left hand object till it is seen through the transparent part of the horizon-glass, and you will likewise perceive its reflected image in the silvered part: then advance the index, and at the same time keep the reflected image in sight, by turning your body and the sextant toward the right hand, till you see the other

object through the transparent part; make them join in the middle of the line of separation of the horizon-glass, and the index will shew the angle on the limb.

This method of observing will be often convenient, because, after a little practice, an angle may be taken more readily this way than with the face of the sextant uppermost; and also because the faintest object may often be found by direct vision through the transparent part of the glass, when it could not be so easily found by reflection.

How to adjust the Horizon-glass for Observation, by a horizontal Line.

To adjust the horizon-glass is, to make it parallel to the index-glass when the pointer of the vernier is placed at 0, or the beginning of the degrees. This is the most important adjustment of all for an observer; and should be done every time one begins to make any observations. The horizontal lines fittest for this operation are, the horizon of the sea, and the roof, or upper part of the wall of a large well-built house above half a mile distant.

To adjust by the horizon of the sea, or any other horizontal line: turn the index, till the pointer of the vernier stands exactly at 0 on the limb, and fix the index there: hold the sextant in a vertical position, and observe whether the edge of the sea-horizon (or other horizontal line) seen in the silvered part of the glass, makes one continued line with the edge of the sea seen in the transparent part: if it does not, and the line appears broken, or one part of it above the other, bring them into one

straight line by turning the adjusting lever at the back of the horizon-glass; then fix the lever by the button-screw in the middle of it, and the horizon-glass is adjusted, or parallel to the index-glass.

To find the error of this adjustment: move the index a little from its place; hold the sextant in a vertical position as before, and turn the index till the line of the sea-horizon (or other horizontal line) seen in the silvered part of the horizon-glass joins exactly in one straight line with the edge of the sea seen through the transparent part: then the number of minutes and seconds by which 0 on the vernier differs from 0 on the limb, is the error of the adjustment; and must be allowed for in all the angles taken with this adjustment, in the following manner: If the pointer, or 0 on the vernier, stands on the quadrantal arc, or to the left hand of 0 on the limb, then this error is to be subtracted from the angle shewn on the limb by the index; but if 0 on the vernier stands on the arc of excess, or to the right of 0 on the limb, then the error of the adjustment is to be added to the angle shewn on the limb. In reading off the minutes and seconds, on the arc of excess, take their compliment to 30, in place of what the vernier shews.

When the index is moved from you, to bring an image and object together, it will give a different angle from that same angle taken by moving the index towards you. This is occasioned by the index bending in not turning freely on its centre, or starting a little by the pressure of the hand. To correct this, when great accuracy is required, take the angles two or three times over each way, and the medium of all will be the true angle. To prevent it, the index should be of a substance not apt to

bend, and as broad as the sextant will conveniently admit of; it should move quite easy round the centre; but by means of a spring and screw, should compress the arc of the instrument with requisite firmness, and so as to slide smoothly along it when moved.

To adjust the Horizon-glass by the Sun.

Set the pointer of the vernier to 0 on the limb; raise the telescope, or tube, as high as the middle of the transparent part of the horizon-glass; use smoked, or red glasses to defend the eye, and slip a skreen of paste-board nine inches square, with a hole in the middle, over the end of the telescope, to keep the face from being scorched; then view the sun and its image both through the transparent part of the horizon-glass, and by the lever make them cover each other exactly, so as to appear as one object; screw the lever fast, and the horizon-glass is adjusted.

The adjustment may be made in the same manner by the moon, or a star, without a darkening-glass or skreen.

To find the error of this adjustment; move the index a little from its former place at 0; then, by the index, make the two images cover one another exactly as before; and the number of minutes and seconds by which the pointer of the vernier differs from 0 on the limb, is the error of adjustment, and must be allowed for as above.

A more correct method of adjusting by the Sun.

Set the pointer of the vernier to 0 on the limb; and with the darkening glass and skreen to defend your eye and face, measure the diameter of the sun, first on the quadrantal arc, and next on the arc of excess; or with the index, first on the left hand of 0 on the limb, and then on the right of it. If these two diameters are equal, then is the horizon-glass adjusted: if the diameters are unequal, turn the horizon-lever a little one way or the other, till upon measuring them again they are found equal. But it is equivalent, and often more correct and convenient, when the error is small, to mark the difference between the two diameters, and make allowance of half that difference in each angle observed on the limb, adding or subtracting it as before directed. In reading off the odd minutes and seconds on the arc of excess, remember to take the compliment to 30 of what the vernier shews.

The diameter of the sun is measured thus. Set the pointer of the vernier at 0, and by the index, bring the reflected image in external contact with the direct image; note the degrees, minutes, and seconds of the pointer from 0; then bring them in contact on the opposite side of the direct image, and note the degree, minutes, and seconds; see whether the diameter on the quadrantal arc, or the diameter on the arc of excess, is greatest; mark the difference, and reserve the half of it for use, if you do not correct the error by the lever.

The adjustment may be made pretty exact by taking

two diameters of a straight-sided land-object in the same manner.

How to adjust the Sextant by any remarkable Land-object at a Distance.

It will often happen on a survey, that no straight line, either vertical or horizontal, nor any celestial body, can be seen to adjust by. In this case, select for the purpose some small remarkable object at a distance, as the tapering top of a hill, rock, small hummock, chimney-head, or detached branch of a tree. To adjust by such an object, set the index to 0; hold the sextant horizontal; view the object through the transparent part, and its image in the reflecting part of the horizon-glass; and by the lever, bring the image directly below the object in the middle of the line of separation, so as neither appears to one hand of the other, but that if a vertical line is imagined to be drawn through any point of the image, it would pass through that same point of the object; then screw the lever fast, and the horizon-glass is adjusted. This method of adjusting may be sufficient for common purposes, but is not so exact as the other methods.

How to examine the Angle of the Sextant.

The angle of the sextant, measured by the arc of the limb comprehended between 0 and 120 may be verified, first, by adjusting the glasses as before directed, and then taking, at one station, five or six horizontal angles, comprehending a whole circle round; add these angles

together, and if their sum amounts to 360 degrees exactly, the angle of the sextant and its graduations may be concluded to be exact. The objects that form these angles must be very remarkable and sharp, and all of the same apparent height,* to prevent mistakes in taking them; otherwise inaccuracy in the observer may be imputed to the instrument.

Another way of examining the angle of the sextant is, by setting perpendicular in a level ground three poles, or staves, between four and five feet high, with flags flying at each, so as to form a triangle, whose greatest angle is less than 120 degrees; let the distance of the poles from one another be half a mile, or as much farther as they can be seen distinctly. After adjusting the sextant, take the three angles of the triangle very exactly, keeping the centre of the sextant exactly on the centre of the staff you observe at; and if the sum of the angles amounts precisely to 180 degrees, the sextant may be concluded to be good. The several graduations, and their sub-divisions, may be examined by the vernier, as directed for the theodolite.

How to examine the Accuracy of the two quadrantal Radii of Astronomical Quadrant.

PLATE I. FIG. 18.

1. On a level line set two poles, A and B, perpendicular in the ground, about a mile, or more, from one

* This favorable position and combination of objects, can seldom be obtained.

another; let a flag fly at the top of each, to render them more conspicuous; and let the poles appear on some remarkable sharp part (C) of a remote object; the farther the object is, the better.

2. Put the quadrant on its stand at B, with its plane set horizontal, by a spirit level laid on it, and its centre right above the hole where the pole stood, and in the position as at b. If the stand cannot support it horizontally, lay the quadrant steady on a firm table. Then, with the index at 0 deg. look through the telescope, and turn the quadrant till you see the object C exactly at the intersection of the wires. Let the quadrant be kept firm in that position, and turn the index to 90, and observe through the telescope what object appears at the intersection of the wires; and if nothing remarkable, set up a pole and flag D, in that direction, as far off as can be seen.

3. With the index at 90, turn the quadrant on its centre a quarter round, till it is in the position a, so that you see the pole A through the telescope at the intersection of the wires. Then keeping the quadrant firm in that position, turn the index to 0 deg. and if through the telescope you see the object D, exactly at the intersection of the wires, then the two radii of the quadrant, drawn through 0 and 90, are precisely at right angles, and the axis of the telescope parallel to the radius it is fastened to. If the object D, does not appear exactly at the intersection of the wires, but on one side of it; then half its distance from the intersection is the error of the quadrant; and is either in the angle of the two radii, or in the axis of the telescope, which is a line supposed drawn through the centre of the eye-glass and

the intersection of the cross-wires. It is much more likely to be in the telescope than in the quadrantal radii; and therefore to correct it, the wires in the telescope must be shifted by their proper screws, half the distance between the object and intersection of the wires; and the quadrant examined over again as before. If it is not right on this second examination, correct the wire over again by the screws, and examine the quadrant a third time; and if the object D, and the intersection of the wires still disagree, conclude the error to be in the angle of the radii between 0 and 90. Mark how much that error is, and by it correct all future observations. This method is liable to a little inaccuracy, if the plane of the quadrant is not made exactly horizontal.

To examine and adjust an Astronomical Quadrant.

1. Set up the quadrant on the margin of a lake, close to the edge of the water, so that the centre of the quadrant is just over the water: (or, in a well-sheltered bay when the sea is smooth, right over the high-water mark) set one side of the quadrant perpendicular by the plummet; and measure the height of the centre of the telescope above the surface of the water (or above the high-water mark) in inches and parts of an inch.

2. About half a mile off (but not above a mile) set a pole straight up in the edge of the lake (or on the high-water mark) with a conspicuous sliding cross-piece on it: let one edge of the cross-piece be set as high above the surface of the water (or high-water mark) as the centre of the telescope was, and about 2 inches higher, if the distance of the stations is ½ a mile; 4½ inches

higher if ¾ of a mile, and 8 inches higher if their distance is one mile; for the height of the tangent above the surface of the earth in these arcs.*

3. Set the index of the quadrant to 0 degrees, or exactly to the horizontal radius, and direct the telescope to the pole: if, while the vertical wire runs along the pole, the horizontal wire runs exactly along that edge of the cross-piece which was set as high as the centre of the telescope, the quadrant and telescope are right. If the horizontal wire does not coincide in that manner, shift it by the proper screws till it does so, and the quadrantal angle and axis of the telescope will then be right.

The screws for shifting the wires are on the telescope, at the common focus of the object and eye-glass. If you see the edge of the cross-piece below the horizontal wire, ease the upper screw, and screw in the lower one;†

* The height of a tangent to the extremity of any small arc above the other extremity of that arc, is a third proportional to the diameter of the earth and the length of the arc. For, imagine a diameter drawn to one end of the arc, and produced till it meets a tangent drawn to the other end; then, by Eucl. b. III. prop. 36. the rectangle of the diameter (in this case the same with the secant) and produced part is equal to the square of the tangent. If the diameter is called d, the produced part p, the tangent t, and the arc a; the two equal rectangles will be expressed thus $d \times p = t \times t$: and because a small arc and its tangent are so nearly equal, that one may be assumed for the other; in place of $t \times t$, substitute $a \times a$, and it will be $d \times p = a \times a$. When two rectangles are equal, their sides are reciprocally proportional (Eucl. b. VI. prop. 14.) therefore $d : a :: a : p$; that is, as the diameter is to any small arc, suppose one minute, so is that arc to the height of the tangent above it. By this proportion the numbers in the text were found, supposing the diameter of the earth to be 41798117 feet.

† As the telescope inverts objects, what appears below the hori-

if the edge of the cross-piece appears above the horizontal wire, ease the lower screw, and screw in the upper one, till the horizontal wire coincides with the edge of the cross-piece; and the axis of the telescope is then right.

This adjustment may be verified, by placing the quadrant where the pole stood, and the pole where the centre of the quadrant was, and the axis of the telescope, and the edge of the cross-piece, at the same height above the surface of the water as at first; then make the radius of the quadrant perpendicular by the plummet, set the index to 0 degrees, look through the telescope, and if the horizontal wire and edge of the cross-piece coincide again, the angle of the quadrant and axis of the telescope are certainly right. If any difference is perceived, move the horizontal wire, by its screws, half that difference, and it will be rectified.

The graduations of a quadrant may be examined in the same manner as those of the theodolite above-mentioned, by applying the vernier to the several degrees, and parts of a degree.

How to examine a Magnetic Needle.

First set the box level, by raising, or lowering, one side of it, till each end of the needle, when it plays from side to side, runs along the circumference of the gra-

zontal wire is really above it, and therefore, screwing in the lower screw, which pushes up the wire, will bring it nearer the object. The contrary is true of the upper screw.

duated circle in the box; and both ends of the needle, when at rest, point equally high on it. Then turn the box gently, till one end of the needle rests at the beginning of the graduations exactly, and if the other end points exactly to 180°, the opposite graduation, the needle is so far right centered. Next turn the box 90 degrees round, and if the ends of the needle then point precisely to the opposite degrees, then the needle is truly centered. Each degree may be tried in the same manner.

Observe likewise if the cap in the centre of the needle is small and tapering in the inside, or large and spherical: if it is a portion of a large sphere, the point of the steeple will be apt to rest on different parts of it, and by that means vary the centre of the needle, and cause it to point wrong.

CHAP. V.

Meridional Problems by the Stars.

How to fix a Meridian Line by a Star, when it can be seen at its greatest Elongation on each Side of the Pole.

PROVIDE two plummets: let one hang from a fixt point; let the other hang over a rod, supported horizontally about 6 or 8 feet above the ground, or floor in a house, and so as to slide occasionally along the rod: let the moveable plummet hang four or five feet north-

ward of the fixt one. Some time before the star is at its greatest elongation, follow it, in its motion, with the moveable plummet, so as a person a little behind the fixt plummet may always see both in one, and just touching the star. When the star becomes stationary, or moves not beyond the moveable plummet, set up a light on a staff, by signals, about half a mile off, precisely in the direction of both the plummets. Near twelve hours afterward, when the star approaches its greatest elongation on the other side of the pole, with your eye a foot or two behind the fixt plummet, follow it with the moveable plummet, till you perceive it stationary as before: mark the direction of the plummets then by a light put up precisely in that line: take the angle between the places of the two lights with *Hadley's* quadrant, or a good theodolite, the centre of the instrument at the fixt plummet; and a pole set up in daylight, bissecting that angle, will be exactly in the meridian seen from the fixt plummet.

This observation will be more accurate, if the eye is steadied by viewing the plumb-lines and star through a small slit in a plate of brass placed upright on a stool, or on the top of a chair-back.

This problem depends on no former observations whatever: and as there is nothing in the operation, or instruments, to affect its accuracy, but what any one may easily guard against, it may be reckoned the surest foundation for all subsequent celestial observations that require an exact meridian line. The only disadvantage is, that it cannot be performed but in winter, and when the stars may be seen for 12 hours together, which requires the night to be about 15 hours long.

To fix a Meridian line by two circumpolar Stars that have the same right ascension; or differ precisely 180 *degrees.*

Pitch on two stars that do not set, and whose right ascensions are the same, or exactly 180 degrees different; take them in the same vertical circle by a plumb-line, and at the same time let a light be set up in that direction, half a mile, or a mile off, and the light and plummet will be exactly in the meridian.

In order to place a distant light exactly in the direction of the plumb-line and stars, proceed in the following manner. Any night before the observation is to be made, when the two stars appear to the eye to be in a vertical position, set up a staff in the place near which you would have the plummet to hang; and placing your eye at that staff, direct an assistant to set another staff upright in the ground, 30 or 40 yards off, as near in a line with the stars as you can. Next day, set the two staffs in the same places; and in their direction, at the distance of a mile, or half a mile northward, cut a small hole in the ground, for the place where the light is to stand at night; and mark the hole so, that a person sent there at night may find it.

Then chuse a calm night, if the observation is to be made without doors; (if it is moon-light, so much the better) and half an hour before the stars appear near the same vertical, have a lighted lanthorn ready tied to the top of a pole, and set upright in the distant hole marked for it the night before; and the light will then be very near the meridian, seen from the place marked for the plummet. At the same time, let another pole, or rod, 6 or 8 feet long, be supported horizontally where the

plummet is to hang, 6 or 7 feet from the ground, and hang the plumb-line over it, so as to slip easily along it either to one hand or the other, as there may be occasion (or tie a staff firmly across the top of a pole 6 or 7 feet long; fix the pole in the ground, and make the plumb-line hang over the cross-staff.) Let the weight at the end of the line be pretty heavy, and swing in a tub of water, so as it may not shake by a small motion of the air. Then shift the plumb-line to one hand, or the other, till one side of the line, when at rest, cuts the star which is nearest the pole of the world, and the middle of the light together: as that star moves, continue moving the plumb-line along the rod, so as to keep it always on the light and star, till the other star comes to the same side of the plumb-line also; and then the plummet and light will be exactly in the meridian.

There are not two remarkable stars near the north pole, with the same right-ascension precisely, or just a semi-circle's difference: but there are three stars that are very nearly so, viz. the pole-star, ϵ in Ursa Major, and γ in Casiopeia. If either of the two last, particularly ϵ, are taken in the same vertical with the pole-star, they will then be so very near the meridian, that no greater exactness need be desired for any purpose in surveying. At London γ is about 1′ west of the meridian then; and ϵ much nearer it, on the west side likewise. Stars towards the south pole proper for this observation are, γ, in the head of the cross; and α, in the foot of the cross; and α, in the head of the Phenix.

How to find a Meridian-Line by a circumpolar Star, when it is at its greatest Azimuth from the Pole.

By a circumpolar star is here meant, a star whose distance from the pole is less than either the latitude, or co-latitude, of the place of observation.

1. Find the latitude of the place, in which you are to observe the star.

2. Pitch on a star whose declination is known, and calculate its greatest azimuth from the north, or elevated pole.

3. Find at what time it will be on the meridian the afternoon you are to observe; so as to judge about what time to begin the observation.

4. Prepare two plummets, and follow the star with one of them, (as already directed) until the star is stationary, then set up a light half a mile, or a mile off, in the direction of the plumb-line and star, and mark the place of the light in the ground, and also of the plumb-line.

5. Next day set up a pole where the light stood, with a flag flying at it. Then with a theodolite, or *Hadley*'s sextant, set the index to the degree and minute of the star's azimuth from the north, found before; direct, by waving to one hand or the other, an assistant to set up a staff on the same side of the pole with a flag, as the pole of the world was from it, so as at the plumb-line these two lines may make an angle equal to the azimuth

of the star, and the plumb-line and staff will then be in the meridian.

When any star is descending, it is on the west side of the pole of the world; while it ascends, it is on the east side of it.

The pole of the world is always between the pole-star and Ursa Major: so that when Ursa Major is west or east of the pole-star, the pole of the world is west or east of it likewise.

To find the greatest azimuth of a circumpolar star from the meridian, use the following proportion.

Cosine of the lat. : R :: S. of the star's distance from the pole: S. of its greatest azimuth.

The greatest azimuth is when the star is below the horizontal diameter of its diurnal circle, as many degrees (measured on that circle) as is equal to the star's polar distance.

On the north side of the equator, the pole-star is the most convenient for this observation; for the time when it is at its greatest elongation from the pole, may be known sufficiently near by the eye; by observing when ε in Ursa Major and γ in Casiopeia appear to be in a horizontal line, or parallel to the horizon; for that is nearly the time. Or, the time may be found more precisely, by making fast a small piece of wood along the plumb-line when extended, with a cross-piece at right angles to the top of the upright piece, like the letter T; when the plummet is at rest, and both stars are seen touching the upper edge of the cross-piece, then they are both horizontal: another conveniency in making use of this star is, that it changes its azimuth much slower

than other stars, and therefore affords more time to take its direction exact.

On the south side of the equator, the head of the cross is the most convenient for this observation, being nearest to the south pole; and the time of its coming to the horizontal diameter of the diurnal circle, when it appears in a horizontal line with the foot of the Cross, or the head of the Phenix; and its greatest azimuth is one hour sixteen minutes before or after that, as it is east or west of the meridian. The pole of the world, is between the head of the Cross and the last of these stars.

To find when any star will come to the meridian, in either hemisphere, south or north. Take the star's right ascension, in time, from the most correct tables, also the sun's right ascension for the day and place proposed; their difference will shew the difference between their times of coming on the meridian in the south, or between the pole and zenith; which will be after noon if the sun's right ascension is least, but before noon if greatest. Eleven hours 58 minutes 2 seconds after the star has been on the meridian above the pole, it will come to the meridian north of it, or below the pole.

Given the latitude of the place, and the declinations and right ascensions of two Stars in the same vertical line; to find the horizontal distance of that vertical from the meridian; the time one of the Stars will take to come from the vertical to the meridian; and the precise time of the observation.

CASE I.

When the two Stars are northward of the Zenith.

PLATE I. FIG. 19.

Let Z P O be a meridian, Z the place of observation, H O its horizon, and Z V a vertical circle passing through two known stars, s and S; P the pole, E Q the equator, P s D and P S d circles of declination (or right ascension) passing through these stars: then is Z P the co-latitude of the place, P s and P S the co-declination of the two stars respectively, and the angle s P S the nearest distance of their circles of right ascension; V O the arc of the horizon between the vertical circle and the meridian; and d Q the arc of the equator between the star S and the meridian. When, in the triangle P Z s, the angles P Z s and s P Z (measured by the arcs V O and D E) are found, the problem is solved.

SOLUTION.

1. Begin with the oblique-angled triangle P s S; in which are given two sides P s and P S, the co-declinations of the two stars respectively, and the included angle S P s; which angle is the difference of their right ascensions when it is less than 180 degrees; but if their difference is more than 180°, then the angle S P s is equal to the lesser right ascension added to what the greater wants of 360 degrees. From hence find the angle S s P by the following proportions.

As radius
Is to the co-sine of the given angle (S P s)

So is the tan. of the side opposite the required ang. (P s)
To the tan. of an angle, which call M.

M. is like the side opposite the angle sought, if the given angle is acute; but unlike that side, if the given angle is obtuse.*

Take the difference between the side (P S) adjacent the required angle and M; call it N. Then,

Sine N : sine M. :: tan. of the given angle (S P s): tan. of the required angle (S s P) which is like the given angle if M is less than the side (P S) adjacent to the required angle; but unlike the given angle if M is greater than P S.

2. Next in the oblique-angled triangle P s Z there are given two sides, P Z the co-latitude of the place of observation; P s, the co-declination of the star s; and the angle P s Z opposite to one of them, which is the supplement of P s S (last found) to 180°: from thence find the angle P Z s opposite the other side, by the following proportion.

Sine of P Z : *sine* P s Z :: *sine* P s : *sine* P Z s. *either acute or obtuse.*

This angle P Z s, measured by the arc of the horizon V O, is therefore equal to the horizontal distance of the

* Two arcs, or angles, are said to be like, or of the same kind, when both are less than 90°, or both more than 90°: but are said to be unlike, when one is greater and the other less than 90°: and are made like, or unlike, to another, by taking the supplement to 180° of the arc, or angle, produced in the proportion, in place of what the proportion brings out.

vertical of the two stars from the meridian. Let the direction of the vertical be taken by a plumb-line and distant light, as before directed, or by two plumb-lines, and marked on the ground. Next day, the degrees and minutes in the arc V O may be added to it by *Hadley's* quadrant, and a pole set up there, which will be in the direction of the meridian from the plumb-line.

3. Last, in the same triangle P s Z, find the angle s P Z between the given sides, by the following proportions.

As Radius
Is to the tan. of the given angle (P s Z)
So is the co. s. of the adjacent side (P s)
To the co. t. of M.

M is acute, if the given angle and its adjacent side are like; but obtuse, if the given angle and the adjacent side are unlike.

As the co. t. of the side adjacent to the given angle (P s)
Is to the co. t. of the other side (P Z)
So is the co. s. M
To the co. s. of an angle, which call N.

N is like the side opposite the given angle, if that angle is acute; but unlike the side opposite the given angle, if that angle is obtuse.

Then the required angle s P Z is either equal to the sum or difference of M and N, as the given sides are like or unlike.

The angle s P Z, thus found, added to s P S, and their sum subtracted from 180° will leave the angle d P Q, or the arc d Q that measures it, which is the arc of the equator the star S must pass over in coming from the

vertical, Z V, to the meridian. Which converted into time, and measured by a clock, or watch, beginning to reckon the precise moment that a plumb-line cuts both stars, will shew the hour, minute, and second that S is on the meridian.

Find, by the right ascension of the star and sun, at what time that star should come to the meridian in the north the night of the observation; subtract from it the time the star takes from the vertical to the meridian, and the remainder, corrected by the sun's equation, will be the time when the two stars were in the same vertical.

The lowest of the two stars comes soonest to the meridian below the pole; the highest of them comes soonest to that part of the meridian which is above the pole.

The nearer in time one of the stars is to the meridian when the observation is made, it is the better: for then an ordinary watch will serve to measure the time sufficiently exact. It is still more advantageous if one of them is above the pole when the other is below it.

The nearer one of the stars is to the pole, and the farther the other is from it, the more exact will this observation be; because the change of the vertical will be the sooner perceived. For this reason, in north latitudes, stars northward of the zenith are preferable to those that are southward of it.

If the two stars are past the meridian when they are observed in the same vertical, then the arc d Q gives the time S took to come from the meridian to the vertical; and must be added to the time when that star was

on the meridian to give the time of the observation: and the arc of the horizon V O must be marked on the ground on the side of the vertical, contrary to what it would have been in the foregoing supposition; that is, eastward *below* the pole, and westward *above* it.

When two stars come to the vertical line near the meridian, it may be difficult to judge on which side of it they are at that time; for determining this, the following rules may serve.

The right ascension of two stars may be either each *less*, or each *more* than 180 degrees, or *one more* and the *other less.*

When the right ascension of each of the stars is *either less, or more,* than 180 degrees, they will come to the same vertical on the *east side* of the meridian when the star with the *greatest* right ascension is the *lowest;* but on the *west* side of the meridian when it is *highest.*

When the right ascension of one of the Stars is more than 180 degrees, and that of the other less.

If the right ascension of the highest is less than 180°, but greater than the excess of the other's right ascension above 180°, then they come to the vertical on the *east side* of the meridian. But if the right ascension of the higher star is less than that excess, they come to the vertical on the *west side* of the meridian.

If the right ascension of the higher star is *more* than 180 degrees, and that excess is *less* than the right ascen-

sion of the lower star, then they come to the vertical on the *west side* of the meridian; but if the excess of the higher star's right ascension above 180° is *more* than the right ascension of the lower star, then they come to the same vertical on the *east side* of the meridian.

CASE II.

When the two Stars are in the same vertical southward of the zenith.

PLATE I. FIG. 20.

When two stars are observed in the same vertical line *southward* of the zenith (or toward the depressed pole) the operations and solutions are nearly the same as in Case I. For let P Z O be a meridian, Z the place of observation, O H its horizon, Z V a vertical circle passing through the two stars S and s; P the pole, Q E the equator, P D and P d two circles of declination (or right ascension) passing through the two stars respectively: then is Z P the co-latitude of the place, P S and P s the co-declination of the two stars respectively, the angle S P s the difference of their right ascensions, the angle S Z Q (or O V the arc of the horizon which measures it) the distance of the vertical from the meridian; the angle s P Z (or D Q which measures it) the arc of the equator, which s must pass over from the vertical to the meridian, which converted into time, and measured by a watch, will shew when s is on the meridian; and subtracted from the calculated time that the star should come to the meridian, will (when corrected by the sun's equation) give the true time of the observation: S P *s* is

the triangle to begin the solution with; and in the triangle s P Z, the angles s P Z and s Z P, when found, will give the solution of the problem, as in Case I. For the supplement of s Z P to 180° is the angle V Z O (or its measure O V) the horizontal distance of the vertical from the meridian; the angle s P Z (or its measure d Q) is the equatorial distance of the vertical from the meridian: for all which the solution in Case I. properly applied, will serve.

On the south side of the zenith, when the *highest* of the two stars has the *least* right ascension, they come to the same vertical on the *east side* of the meridian; but when the highest star has the *greatest* right ascension, they come to the same vertical on the *west* side of the meridian.

MARINE SURVEYING.

PART II.

OF THE PROCEDURE AND OPERATIONS IN SURVEYING SEA-COASTS, ACCORDING TO THEIR VARIOUS CIRCUMSTANCES.

MARINE SURVEYING.

CHAP. I.

How to form a stasimetric scheme of points, by which the distances along the Coast may be determined.

IN order to survey a bay, harbour, river, or any part of the sea coast, with sufficient accuracy and expedition, three principal things are requisite. The first is, to measure a *fundamental base line:* the second, to form a *stasimetric scheme* of points, by which other distances along the coast may be determined: the third is, to delineate on paper, *a figure similar to the proposed coast.* The several methods of finding the length of a base line having been explained in Part I. Chap. III. the next is, how to form the *stasimetric scheme,* which may be done in the following manner.

PLATE II. FIG. I.

1. Having first taken a general view of the place to be surveyed from an eminence, in order to find out proper objects for the scheme, and to plan in your mind the procedure of the survey; pitch on the smooth sand X Y for a fundamental base line, as near the middle of

the harbour, or part to be surveyed, as can be had conveniently: on it measure the straight line X Y with a pole, or chain, as directed Part I. Chap. III. Sec. I.; mark its extremities on the ground, and write the number of poles, or chains, in a field-book prepared for the purpose.

2. At X, with a sextant or theodolite, take the angles, that some of the most conspicuous and sharpest objects (such as steeples, towers, remarkable tops of hills, &c.) on each side of the bay, make with the base-line; as Y X A, Y X B, Y X C, Y X D: and as many other parts along the coast as are distinctly seen, and may be judged convenient. If there are no remarkable objects to be seen that are small enough, cause turrets, or signals, of stone or turf, to be built for that purpose, on the most convenient hills or eminences. Also, at Y, take the angles X Y A, X Y B, &c. then take the bearing of X from Y, with a good magnetic needle and sights: write these angles distinctly in the field-book, together with the name, or description, of each object, and the part of it which you observe, so that they may be known again when needed, or when you go to them: write down likewise, on which side of the base-line each object lies, whether on the right or left, when you look towards the farthest end of it.

3. Near the middle of a sheet of large paper (so as there may be room for laying the protractor, and drawing out the angles) draw the magnetic meridian right up and down. In a convenient part of that line, mark a point X, to represent one extremity of the base-line; and at X, make an angle with the meridian equal to the bearing of Y from it. On that bearing set off the length

of X Y, in miles and parts of a mile, taken from a scale of equal parts, not less than an inch to a mile.

4. From X and Y on the paper, protract carefully, with a large protractor, the several angles written in the field-book; and also calculate trigonometrically the most material distances: judge of the accuracy of the protraction by its agreement with the calculation; and if any of them differ, protract, or calculate these again till the error is corrected, and the points representing the several objects are marked in their proper positions and distances on the paper.

5. To verify these protracted distances, go to any of the objects, as D, take the bearing of X and Y, to find if they agree with the protraction; if so, then take the angle X D Y, add it to the two other angles of that triangle, and if their sum is 180 precisely, then is the position of D exact: there also take the bearings, by the needle, of the other objects; and if their bearings from you agree with their bearings from D on the paper, then your several operations have been rightly performed, and the stasimetric scheme is exact. If the bearing of any of the objects by the needle differs from that of its representative point on the paper, take the angles of that object again from the two extremities of the base-line, and protract them anew, till you discover the error, and find all agree: then, and no sooner, should you proceed to delineate the coast-line on the paper.

NOTE 1. Very high mountains, whose tops are often hid in clouds, are on that account not so convenient objects for a stasimetric scheme, as hills of a more moderate height, which may be seen more frequently.

Note 2. As the use of this stasimetric scheme is for finding the distances of other parts along the coast, by one or other of the longimetrical problems in Part I., great care must be taken to get the distances and positions of the points in it exact: therefore, if any of the objects lie too oblique to the base-line, or be too far from it to have its distance determined precisely; let *one* of the angles only be taken at the base-line, and the other from one of the objects whose distance is ascertained, and makes a more direct intersection with it.

Note 3. If no level sand can be had within the bay, or harbour, for a base-line; then a smooth plane near it, on either side, may be measured, and the distances of the objects on each side of the bay deduced trigonometrically from thence.

Note 4. If no plane fit for a base-line, is in, or near the harbour to be surveyed, and if it is but a few miles in extent; then, the distance of objects properly situated, may be measured by the velocity of sound, (see Part I. Chap. III.) and that distance made a base-line, from which the stasimetric scheme may be deduced.

Note 5. If the bay, or river is above 8 miles in extent, and no sufficient plane in, or near it; then take the latitudes of two remarkable hills, or objects, at least 15 miles asunder, and not above 45 degrees from the meridian; from thence find their distance in miles and parts of a mile (by Prob. II. Part I.) make that the base-line, and from thence calculate and protract the distances in the stasimetric scheme.

The form of a Field-book.

Measured in a straight line on the sand on the south side of the entrance of Great Harbour, from X (at the high-water mark of White Point) to Y (at the edge of the grass on Green Point) 416 chains, 25 links; each chain 60 feet long, and each link 1 foot; amounting to 4½ miles and 408 yards.

At X, (by the needle) Y bears — W. 18½° N.

Angles taken at X, *on the side* X Y. — Y *bearing* W. 18½° N.

			Deg.	Min
Left Side.	a	Signal on Flat Hill - - - - -	45	30
	b	Steeple of New Church, and point of a clay-cliff about a mile beyond it -	18	15
	c	Sharp Rock on Black Head, and 1 small house on with it - - -	2	30
	d	North End of a Bluff Point - -	15	0
	e	Castle Strong, W. Corner - -	21	10
	f	Low Rocky Point N. of that castle -	12	5
	g	E. Gavel of Wood House in one with Y	0	0
Right Side.	h	Shoal Bay, E. point - - -	6	50
	k	Do. do. Western Bluff - - -	3	25
	l	White Cliff - - - - -	11	20
	m	Sand Down - - - - -	27	5
	n	Creek Mouth - - - - -	36	8

Angles taken at Y, *on the Side* X Y, X *bearing* E. 18½° S.

Side		Object	Deg.	Min.
Right Side.	a	Signal on Flat Hill - - - - -	10	30
	b	Steeple of New Church - - -	63	50
	c	Sharp Rock on Black Head - -	171	20
	d	Bluff Point - - - - - -	148	50
	e	Castle Strong W. corner - - -	145	10
	f	Low Rocky Point N. of that castle -	157	45
Left Side.		Hillock near Sandy Point, and point of a high rocky cliff, supposed three miles off - - - - - -	38	27
		W. Gavel of Cot House - - -	134	15
		Ben-more Hill, the westernmost and most tapering top; also a breaking Shoal about a league off - -	165	51
	h	Shoal Bay E. point - - -	166	0
	k	Do. do. western bluff - - -	174	10
	l	White Cliff - - - - - -	162	55
	m	Sand Down - - - - - -	141	40
	n	Creek Mouth - - - - - -	130	10

In taking the angles at Y, take the same part of each object which was done at X; and before protracting, prefix the same small letters to the names of the same places, or objects. These letters serve to mark the several directions of the angles in protracting, so that the intersection of the corresponding sides may be more readily found on the paper.

CHAP. II.

Examples of the procedure in surveying Sea-coasts, under the most common Circumstances.

Example I.

How to survey a Bay, Harbour, or River, and to delineate the Coast-Line on Paper.

Plate II. Fig. 1.

IN this operation it will save much time and trouble, if the surveyor has been accustomed to estimate, by the eye, small distances, such as a mile, half a mile, a quarter, and 200 yards, something near the truth. This dexterity may be acquired, sufficiently exact for the purpose it is to serve, by frequently viewing with attention such distances when actually measured, and comparing other distances with the ideas of these in the memory; especially when the nature of the ground compared, and the darkness, or clearness of the weather, are nearly the same.

First, form a stasimetric scheme of points, as above delineated; and that the points may be more distinct, and the paper not incumbered with a number of unnecessary lines, transfer the magnetic meridian, and the several ascertained points in the scheme, to a convenient part of a large sheet of clean paper (or of several sheets pasted together) so as there may be room on it to contain and protract the rest of the coast: then draw a small black-lead circle, or dots, round each of these points to

render them more apparent, and write near each the name of the object it represents. Next, divide the paper all over into small squares, by black-lead lines about an inch asunder, drawn parallel, and perpendicular, to the magnetic meridian. These parallels serve for laying the protractor north or south, east or west by the eye readily, in any point from whence a magnetic bearing is to be drawn. If one direction only of any other sharp object has been taken, draw it out likewise from the station at which it was observed; and write its name along that line, with a (D:) before it, to signify that it is the direction of that object; or an (L:) if it is the limit of any head, or point. If Fig. 1. is the harbour, the new stasimetric scheme will then look as in Plate II. Fig. 2.

In the next place; with this scheme of points and angular directions, go to any station on the shore whose place is marked on the paper, as X, and there, with a good magnetic needle, take the bearing of a from it; draw it on the scheme from X, the corresponding point there, and judge its distance as near as you can; take that distance in your compasses from the same scale of equal parts by which X Y was laid down; mark its extent from X on the bearing-line, and trace with the point of your compasses on the paper the curvature of the coast between X and a, as exactly as you can: take also the bearing of b from X, and draw it on the paper. Next go to Y; at it take by the needle the direction of b, draw it on the paper, and it will intersect the other direction taken at X in b; trace the apparent curvature between Y and b: take also the bearing of a, and that drawn out will intersect its former direction from X in the true place of a; sketch the curvature between a and

b, and black-lead the whole from Y to X, which will finish that part of the coast-line. Next take the bearing of c from Y, draw it out on the scheme, and on that line mark its judged distance, and with your compasses trace on the paper the curvature of the coast between them. At c intersect that bearing by the bearing of C, or D, and correct that part of the curvature which is next c, and pencil the tracing between c and Y: then take the bearing of d, draw it on the scheme, mark its judged distance on that bearing, and trace the curvature between c and d with your compasses: take also the bearing of g, draw it on the paper, and mark it so as to be found again when you come to g. At d take the bearing of D, to intersect the former bearing from c in d; correct the curvature between them, and pencil it. Take also the bearing of c, mark its judged distance, and trace the curve between them: at e, by the bearing of D, intersect its bearing from d, correct the curvature, and pencil as before. In the same manner, fix by magnetic intersections the points f and g, and trace the curvature and windings of the coast between them. But because the point h is more in the way of ships sailing up the bay than any of the former parts between c and h, its position should be determined in the exactest manner. Therefore at h, with a theodolite, or Hadley's sextant, take the angles A h B, B h C, and from thence (by Longim. Prob. III. Part I.) find the point of station h; which will give its true place independent of the former stations. Next draw the magnetic bearings of the points i, k, l, m, n; intersect these lines by the bearing of C taken at each of them; delineate the curvatures between them, and the coast-line of the bay will be so far finished. In this manner proceed along the whole shore of the harbour, *first* fixing the place you stand on

by intersections of the needle; but at proper distances, and parts that are of consequence to navigation, let these be determined by two angles taken more accurately, and protracted by one or other of the longimetrical problems in Part I.

If all the stations along a coast are determined by magnetic intersections, the natural defects of the needle will, in time, produce sensible errors. If all the stations are determined by the angles between three objects (according to a Longim. prob. in Part I.) though that in strictness is the most exact way, yet it would be much more tedious, confuse the draught with a multitude of intersecting lines and arches, and answer no purpose in navigation better, nor in any respect render a draught more serviceable than if only some of the most material stations, and principal projections of the coast here and there, were so determined, and the intermediate parts found by intersections of the needle.

If, at any part of the coast, two, or three of the necessary stasimetric objects cannot be seen, because of interposing hills; or are not in a convenient position for determining the station wanted; go to some eminence near that station, from whence the objects may be seen more advantageously, find the point of station on that eminence by one of the longimetrical problems in Part I. protract it on the paper, and by it, with one or two of the other objects, find the station wanted on the coast.

If at any station, the angle between two hills, or objects, is wanted, but interposing eminences, or other objects, hinder one of them to be seen; in that case, go to the top of the interposing eminence till you see the

hill and station from it, and exactly in a line with them set up two poles; and the angle between any of the poles and the other object, or hill, taken at the station, will be the same with the angle between the two hills. Thus may the angle, or bearing of any objects, be found from a place where they cannot be seen, which will be necessary or convenient on many occasions.

Mountains, rocks, and stones, are frequently endowed with a magnetical quality, which disturbs the polarity of a needle, and, if not adverted to, will create much trouble. To prevent this, it is necessary to carry two needles always along with you, and to set both down at each station, beyond the reach of each others influence; if they both agree in the bearing of the same remote object, it may be concluded that they are not disturbed by magnetism.

At every station take care to insert, and to distinguish in the draught, the appearances and nature of the neighbouring parts of the coast, such as rocky cliffs, grassy cliffs, sandy shores, rocky shores, &c. and remarkable hills, houses, trees, &c. but especially such as may serve for land-marks to direct ships through channels, or to avoid rocks and shoals. The hills and houses ought to be represented in such prospects as will enable sailors to know them when they are seen from the sea, without regarding much the diversity of perspective which this will occasion in the same draught; for uniformity ought always to give place to utility.

If the survey is to be carried on without the harbour, along the coast on each side, it may be continued on the foundation laid before, and by the same procedure, to

as great a distance as the stasimetric objects can be easily seen, and while the angles they subtend are not too acute to be relied on, or protracted with certainty. Before this happens, it will be necessary to pitch on other stasimetric objects as you go along the coast, to determine their distances carefully by one of the longimetric problems, and make these supply the places of such of the former as are too remote; and by these last, others still farther along may be determined, to serve the same purpose to a greater extent. But let it be adverted here, that if too long a series of triangles is continued from one base-line, the unavoidable imperfection of instruments, observations, and protraction, will, in time, produce very sensible errors: therefore it is advisable to break off the series at proper distances, and to lay a new foundation by measuring another base-line, and proceeding by it as at first. When this is done, remember that two objects, or points, in the first draught, at a proper distance and position, be likewise included in the second; that by these points the two draughts may be easily joined together, and make one map when it is needful.

When the coast of a bay, harbour, or river is surveyed, or any considerable part of it, then find the distance and extent of all the rocks, sand-banks, and shoals that lie near it; observe leading marks to them; and land-marks for avoiding them; sound the depth of the water to, round, and from each; and also along the shore, as near to it as vessels can, or ought to sail. Also sound particularly the several places of anchorage, and take land-marks, or bearings for finding them, and insert an anchor in the proper part of the draught, for pointing them out more readily. Insert also the time of high

water on the days of new and full moon, the ordinary rise of spring and neap tides, the direction of the stream of flood, and what irregularities may be observed in these, both within and without the harbour.

Example II.

How to survey an Island.

Case 1. *If the island has several conspicuous hills in it:* first measure a base-line on a plane, or by two latitudes. 2. On that base-line form a stasimetric scheme of points, as was explained in Part II. Chap. I. § 3. Survey and delineate the coast of the island as directed for a harbour in the last example.

Plate III. Fig. I.

Case 2. *If the island is but a few miles long, and has but one remarkable hill, or object in it,* as A, which may be seen from all, or most parts of the coast: by one of the longimetrical problems in Part I. find the distance of any of the points, or heads, in the island (suppose B) from A. At B, take the magnetic bearing of A, in degrees and parts of a degree; lay down that distance on paper according to the bearing observed; and draw black-lead lines all over the paper, parallel and perpendicular to the magnetic meridian, about an inch from one another. At B, with a theodolite, or Hadley's sextant, take the angles A B C and A B F; write them down first in your field-book, and then draw them out from the point B: also take the bearing of a

and b by the needle; judge of their distances from B; draw them, and sketch the curve of the coast between you and them. Then go to C; there take the angles A C B, A C D; write them down; add A B C and A C B together, subtract their sum from 180°, and the remainder will be the angle B A C; draw that angle from the point A, and it will intersect the side B C in the point of station C. At C, take the bearing of a, and that will intersect its former bearing from B in the corrected place of a; then sketch the curvature between C and B, and pencil it. Next draw the angle A C D; at D, take the angles A D C, A D E; the former added to A C D and subtracted from 180° will leave the angle C A D; draw that angle, and it will intersect the side C D in the station D: sketch and pencil the curve between C and D. Then draw the angle A D E, and the bearing D c; fix the point E by the angle at E, and finish the bay between D and E as before. Go on in this way from point to point round the island, till the whole is finished, remembering to insert remarkable cliffs, houses, or other objects at each station.

If at any point, as G, or H, A cannot be seen; take the direction of A from that point, by setting two poles exactly in a line between them, as mentioned in the preceding example, and take the angle between one of the poles and the other visible object. By such a procedure as this, the circumference of any island will be found to meet on paper, without arbitrary alterations.

When an island is small, and great exactness unnecessary; if the first distance, or base-line, is true, and long enough, all the other distances may be got suffi-

ciently exact by intersections of a good needle, without any other instrument to take the angles.

When one side, or part, of the island, is delineated on the paper, then find the distance and dimensions of the several rocks, sand-banks, shoals, and ledges along it; sound the depth of the water toward and round each of them, and take leading-marks on them, and land-marks for avoiding them: sound also along each part of the coast after it is delineated, as near the shore as vessels can, or ought to sail; and when the whole coast is finished, sound round it a mile, or more, from the land.

Example III.

How to proceed in surveying an extensive Coast.

Case 1. *If the coast extends northward, or southward*, take, carefully, the latitudes of two remarkable hills, or promontories along the coast, as near the true meridian as they can be found, and as far from each other as can be seen distinctly: suppose 20, 30, or 40 miles. From their difference of latitude, bearings and variation of the needle, find their distance in miles and parts of a mile (see P. 1. Prob. 11.) make the chord of that arc, or distance, the base-line, and by it form a stasimetric scheme of points; one, or two of them, representing remarkable and sharp objects. It will be convenient, if one or more of the objects lie off the coast, at sea, as there will be fewer objects to intercept the view of them. When the stasimetric scheme is verified, and transferred to some sheets of clean paper, and a

number of magnetic meridians, and east-and-west lines, drawn over it, then begin to survey and delineate the coast, as directed in Example II.

It will sometimes happen that no proper object is seen from *both* ends of so long a base-line, with which to form a stasimetric scheme; but if any remarkable intermediate object, properly situated, is seen at one end of the base-line, and the other end seen at *that* object, its distance may be found with equal, or rather more accuracy, by taking one angle of the triangle, formed by the base-line and object, at that end of the base-line where it is seen, and the other angle at the object; thence, the third angle is found; and the distance of the object. These three determined distances will be sufficient for finding all other distances between the two extremities of the base-line; and also for determining other stasimetric objects, necessary for continuing the survey far beyond these extremities.

If the latitudes of the two places are correctly determined by observation, a base-line of 30, or 40 miles measured in that manner, is more to be relied on, than such a distance determined by a base-line of 3 or 4 miles measured on a plane: because, planes so long can seldom be found, without some sensible irregularities in them; nor two objects so remote, sharp enough for taking the angles at each end of such a base-line with precision. These two sources of inaccuracy may occasion a greater error in a distance of 20, or 30 miles, than can be supposed when the latitudes are carefully taken with a good sextant.

When the survey has been continued by a train of stasimetric triangles, to a considerable length beyond the

base-line, it will then be proper to discontinue the procedure on that foundation, and to measure a new base-line; either on a level plane, or by two latitudes, as before; taking care to have two determined points common to both draughts, for connecting them into one.

If the instrument with which the angles are taken do not give them minutely enough; or if the objects that form the stasimetric triangles are not sharp enough, errors may be expected, and will undoubtedly become sensible in a long continued series of triangles. How far they have actually taken place in the draught, may be discovered by comparing the observed bearings of distant hills or head-lands, whose positions have been determined by former observations, with their bearings in the draught: or, by comparing such moderate distances as one can judge of by the eye, with their protracted distances on the paper.

When a considerable length of the coast has been surveyed, the soundings marked near it, and all the rocks, shoals, banks, remarkable hills, buildings, groves of trees, and other distinctions of the coast, inserted and expressed in the draught; then sail along it, 6, 8, or 10 leagues from the land, according as it can be seen distinctly; sound the depth of the water, observe the setting of tides and currents, and sketch views of the coast as you sail, inserting in them the names of the most material hills, heads, entries of rivers, harbours, &c. so that seamen may know, by the eye, where the principal places on the coast lie, and how to steer for them.

Case 2. *If the coast to be surveysd extends eastward, or westward;* chuse a remarkable hill, or head,

near the coast, and another hill, or remarkable object up the country, northward or southward; find their distance by measuring a plane, or by the latitudes, and make that distance the base-line; from thence, form a stasimetric scheme of points; and with it, proceed to survey, and delineate the coast as before directed.*

If any part of a coast which extends eastward or westward, is so circumstanced, as neither to have in it, a level plane fit to be measured, nor any hill or remarkable object inland, or lying off the coast, far enough distant to become a base line by taking their latitudes: in that case, build a wall or turret, of earth or stone, on the most conspicuous part of the shore; and another turret inland 3 or 4 miles from it, so large as to be seen 5 or 6 miles off, or farther :† measure the distance of the two turrets by the velocity of sound, and make that a base-line, from whence to determine trigonometrically the distance of the other stations and signals set up along the coast for that purpose: from these last, find other distances; then, if you meet with no plane fit to be measured, nor any remarkable hill or object, at a sufficient distance and position for determining a new base-line by the latitudes; measure a new base-line by sound, and proceed as before. Such a case as this is very rare; but when it happens, a good portable telescope, or spy-glass, will be found convenient, and should be provided accordingly.

* Since the general introduction of marine chronometers, extensive base-lines will be more readily, and accurately measured by them, than by the methods here described, along coasts which extend east or westward.

† Flagstaffs erected with conspicuous flags on them will answer the same purpose.

That the maritime survey of a kingdom, or large tract of continent, may be carried on with expedition and accuracy together, it is necessary one superintendant, or head surveyor, expert in theory and practice, should have two assistants under him, who are capable of executing his orders. *Their* duty is, to conform to his directions diligently and faithfully, as far as can be done; to omit no part of the coast, nor neglect any rocks, shoals, channels, tides, or necessary soundings; to endeavour to get information concerning them from the inhabitants, or pilots, wherever they come, but to insert nothing in their draughts but what has been actually examined by themselves; to keep a daily journal of their operations, observations, and likewise of what information they may receive from others that have not been examined by themselves.

The *head surveyor's* duty is, to plan and direct the procedure of the whole survey; to order the vessel, boats and men on the service, when and where he sees necessary; to chuse proper planes and distances for measuring fundamental base-lines; to see the mensuration, or celestial observations himself; to pitch on proper objects for the stasimetric scheme; and see the angles taken that determine their distances; to inspect the calculations and protraction; to verify the scheme when protracted; to cause a clean copy of it to be made for himself, and one for each of the assistants; to send one of them to survey on one side, or towards one end of the base-line; another on the other side or end of it, and himself to remain with the vessel that attends the survey, and to survey in that neighbourhood; to examine their several performances when they return to the vessel; to compare the most material distances in their

draughts with the observations by which they were determined; to point out mistakes, or defects, and cause them to be corrected; to insert the several observations, measurements, descriptions, and sailing directions, regularly in a book; to join the several parts of the coast, as they are completed, into one draught; and when that is of sufficient extent, to cause a clean, distinct copy to be made of it aboard: then, to sail in the vessel to the next stationary harbour; to cause soundings, and useful views of the coast to be taken by the way, and such remarkable objects on land to be inserted in the draught, as may have been omitted by the assistants. There will be no great advantage in having more than two assistants under one superintendant; for this would often occasion either delays in waiting for one or other of them before the scene of operation could be shifted; or else a superficial inspection of their performances.

Toward the end of harvest, when the days decrease, and bad weather may be frequently expected, a survey will be sooner despatched, if the examination of shoals and sand-banks, that lie at a distance from the land, be postponed till the end of the following spring; and the survey of the coast *only*, and the soundings near it, or of rivers and narrow arms of the sea, are taken in the winter and spring seasons.

Though generally preferable, that a considerable part of a coast be surveyed before the soundings are taken near it, yet often both may be done together with sufficient exactness; by making an eye-sketch of the small bays and windings of the coast, as you go from point to point in a boat to determine their distances, taking the soundings by the way, and inserting them in the cor-

responding parts of the sketch as near as you can judge; and at the same time marking down the direction of the boat, or on what object her head is kept, in sailing or rowing from place to place; and the bearing of one or two objects when the principal soundings were taken: when that part of the coast is surveyed, these soundings may then be transferred to the draught by the bearings, and by the direction in which the boat was steered. If any shoals are met with, take marks on them, or two contiguous angles by *Hadley's* sextant, and examine them more particularly afterward, when the survey of the adjacent part of the coast is finished.

Example IV.

How to proceed in surveying a large Cluster of Islands.

Plate III. Fig. 2.

Pitch on two remarkable tapering hills, rocks, or other objects, at a competent distance, as A and B, as near north, or south, of each other as you can; and if a line connecting them extend about the middle of the cluster, it will be preferable. Find the distance of A and B, either by measuring a straight line on a plane, or by their latitudes, and make that the base-line: select amongst the other islands one, or more, remarkable objects on each side of the base-line; as C and D; of these form and verify a stasimetric scheme, as before directed. With this scheme go to one of the islands, and by Longim. Prob. III. or VI. find the distance of two proper parts, or points in it; make that a base-line for surveying it, as explained in Example II. Do the

same for each island, sounding the channels between them, and observing the setting and strength of the several streams of tide as you pass and repass, taking notes of all, and inserting them in the draught at the time they are observed. When all the islands are surveyed, channels sounded, and every necessary particular inserted in the draught, sail round the whole, between 4 and 10 leagues from the land, sound the depths at this distance, and draw views of the land as you sail along.

If the islands extend northward and southward in a narrow chain, (as in fig. 3.) and are so small, or so uneven, that no sufficient base-line, for the whole, can be measured on either of them; then find by their latitudes the distance of two of the most conspicuous hills, or rocks, that are situated along one side of the cluster, and at a sufficient distance from each other, as A and B. At A and B with *Hadley*'s sextant, or a theodolite, take the angles B A C, B A D, A B C, A B D, and from thence and the side A B, find the places of C and D, any two remarkable objects on the other side of the cluster of islands, which may make the angle B A C and A B D as near to right angles as may be conveniently had. From any three of these points the position of a place on any of the islands may be found; or by intersections of the needle from *two* advantageously situated; and each island delineated separately, as before directed.

CHAP. III.

Examples of the Procedure in surveying Sea-coasts under unfavourable Circumstances.

Example I.

How to survey small Islands that extend East or West in a long narrow Chain.

Plate III. Fig: 4.

IF the islands are small, and extend eastward or westward in a long narrow chain, as fig. 4., no sufficient base-line will be found, either by measuring a plane, or by the latitudes of any two islands: in this case, if the whole extent of the islands is not above 12 or 15 miles, a base-line may be measured by the velocity of sound, a stasimetric scheme protracted from thence for determining the distances of the several islands from one another, and each surveyed separately, as before directed. But if the islands extend above 15 miles in length, such a procedure will not be found sufficiently exact, because a distance cannot be measured by sound with precision: then the following method is more to be relied on.*

Pitch on the most remarkable head, hill, or object, that can be found near the middle of the range of islands,

* Or more correctly by measuring long base-lines by chronometers.

as X: find its latitude, and the variation of the needle there. Then 5 or 6 leagues northward, or southward of X, anchor a vessel, as at V: on a prefixed day, let the latitude of the vessel be observed correctly aboard with *Hadley's* sextant: and at the same time, while the vessel remains steady in the same place, let the angles X V C, X V D, be also taken with *Hadley's* sextant, and as many more angles between them as there are remarkable objects to be seen. About mid-day, while the sun's altitude is taking on board, let one assistant at X find the magnetic bearing of the vessel from thence; and also set up a pole half a mile, or a quarter of a mile from it, exactly in the direction of the vessel; and let the place of the pole be marked on the ground, for future use. Let two other assistants be stationed one at A, and the other at B, 4 or 5 miles on each side of X, and there mark the direction of the vessel from them at mid-day, by poles in the ground as was done at X, so that the angles V X A, V X B, and X B V, X A V, may be taken afterwards at leisure, though the vessel is removed. Find the distance of X V (by Longim. Prob. II.) make that the base-line; on it protract the several angles observed on shore, and aboard, and form a stasimetric scheme of points; in which the stations X, V, A, B, will be determined; and also one direction of C and D, and of the other objects observed at V. From the angles which two, or three of these objects subtend, the distances of the other islands, or of any station in them, may be found, and each island surveyed separately.

If the whole extent of the islands cannot be included in the angles taken at V, the same operations must be

begun again where the former ended, and the rest of the islands surveyed in the same manner.

Example II.

How to proceed in surveying a Coast covered with Wood, or Bushes, so that in-land Objects cannot be seen from the Shore.

Plate III. Fig. 5.

Case 1. *If it is a river, bay, or harbour;* find stasimetric objects on the side opposite to that which you are to delineate, and by these determine your stationary distances by two angles, or by intersections of the magnetic needle.

Case 2. *If it is an open coast, that has no land opposite to it near enough.* 1. Measure a base-line on a plane, or by two latitudes, and from thence find the distances of three remarkable in-land hills, or objects, as Y, R, W, that may be seen from a boat at sea, over the trees; and protract a stasimetric scheme on paper including these three objects. If three remarkable objects cannot be had; two conveniently situated for magnetic intersections along the coast will serve.

2. Prepare eight poles, each with a flag flying at its top for signals; two of the flags white, two red, two yellow, or striped, and two black: the white to represent one of the stasimetric objects, the red another, the yellow or striped a third, as shall be agreed on between the surveyor and his assistant; the black any new stasi-

metric object, whose place may be found necessary to be determined as the survey goes on. Let these colours be written on the stasimetric scheme, each at the point, or object, signified by it; and a copy of the scheme given to the assistant to direct him, and prevent mistakes; together with four of the signals, each of a different colour: the other four are to be carried along with the surveyor.

3. At the first point, or station on shore, whose distance is to be determined, as A, let the surveyor set up a signal of a colour indicating what stasimetric object is to be first observed, as Y, or yellow. Then the assistant must go off in a boat to X, as far from the shore as to see the stasimetric objects over the trees, and take that signal in a line with the object it denotes, keeping the boat steady at anchor in that place, and putting up a signal of the same colour with the surveyor's, to signify that the boat is then in the right place. Let the surveyor then take exactly the magnetic direction of the signal X in the boat; which will also be the direction of his station B from the indicated stasimetric object Y. Let it be drawn out on the paper from its proper point Y, and a signal of another colour R, set up in the place of the former, to signify to the assistant to move the boat till he takes that signal in a line with the object R it represents; and there let the assistant, when ready, put up the like signal at Z, and keep the boat at rest till the surveyor has taken its magnetic direction, and drawn it on the paper; which will intersect the former direction in the point of station A, on the shore.

4. Let the surveyor then take the magnetic direction of the next point of land B, along the shore; draw it on

the paper; estimate its distance by the eye; mark that distance on the line of direction; sketch with the point of his compasses the figure of the curvature of the coast between the two points A and B; take down his signal and wave it, for a sign to the boat to move towards the next point B, on the coast.

5. When the surveyor comes to that point, let him put up a signal W, or white, intimating what stasimetric object the assistant is to take in a line with it; which when he has done, let the assistant set up the like signal in the boat, and keep it steady at anchor till its bearing is taken on the shore; which will intersect the line of direction taken at A the first station, in the corrected distance of B the second station: let the curve of the coast between them be also corrected, and drawn with black-lead, and the coast-line between these two points of land will be finished.

Let the distance of the next point C be determined, and the coast-line sketched, in the same manner; and go on thus till the stasimetric objects can be seen by the surveyor on shore, when there will be no occasion for the boat.

If any of the stasimetric objects shall happen to be hid from the assistant's sight by interposing hills; he must then first set up the surveyor's signal, and above it set up that colour which points out some other stasimetric object that is seen: which when the surveyor observes, he must set up another signal of the same colour above his former one, and then take the direction of the signals in the boat; which will be the direction

of his station from the object represented by the uppermost colour.

When any of the stasimetric objects begins to disappear, or to be too remote, or oblique, for accuracy; then, when the surveyor is at a proper station, the assistant must set up the black signal; to signify that the place of a new stasimetric object is to be determined. The bearing of the black signal taken when the boat is at rest, and drawn out from the surveyor's point of station, in the opposite direction, will be one direction of the new stasimetric object. When the surveyor comes to another station proper for another intersection, the black signal must be set up again by the assistant: and when the boat is at rest, the surveyor is to take its magnetic direction, which will intersect the former in the place of the new stasimetric object; which must continue to be pointed out thereafter by the black signal; till, by agreement, it shall be changed for another colour, that the black colour may be again reserved for another new object, in case it shall be necessary.

As the assistant goes along the shore in the boat, he may at the same time sound the depths of the water, and mark them on an eye-sketch of the coast; or insert them by their bearings from the stations, and other remarkable parts of the coast.

For this manner of surveying, it is necessary that the assistant should be tolerably acquainted with the business, that he may be capable of making a proper choice of objects, and stations for determining their distances.

This method of surveying a coast is sometimes ne-

cessary; and though not so exact as when the angles can be taken with a theodolite, or *Hadley*'s sextant; yet when the stasimetric objects are advantageously situated, and the coast so surveyed is not very extensive, it will be found sufficiently exact for the purposes of navigation. If any particular part of such a coast require great exactness, three assistants, and three boats, should be employed at the same time; each must have the same sort of signals, and they must take the surveyor's signal bearing on three different stasimetric objects at once; by which means he may take the angles between them by a theodolite, or by *Hadley*'s sextant, which will be the same with the angles between the three stasimetric objects; and from thence, may be found the place of station precisely, by Longim. Prob. III.

If there is any stream of tide, or breeze of wind, where the boat lies, it will not be kept steady by one anchor only, but must be made to ride by two, in order to continue in the same place while the surveyor makes his observations on shore.

Example III.

How to survey a Coast, without landing.

Plate IV.

Upon some coasts it is impracticable to land, where a continued swell and surf rolls on the shore; and upon others, the barbarity of the inhabitants renders it unsafe; the survey of such places, must therefore be made

altogether at sea, and it may be done in the following manner.

CASE 1. *If the Coast extends northward and southward.*

1. Find the variation of the compass.

2. Chuse two remarkable tapering hills, or other sharp objects, 20 or 30 miles asunder, and as near the same meridian as they can be had; but so situated, that by sailing along the coast or into a bay, or river, one of them may be taken in a line with the other, so that their mutual bearing may be exactly taken with a good amplitude compass.

3. The variation being found, bring a ship to an anchor right east, or west, from each hill or object, on different days, and there observe their latitudes with *Hadley's* sextant; which will be the latitudes of the two hills respectively.

4. From these latitudes and their bearing, find the distance of the two hills, by Longim. Prob. II. and protract a stasimetric scheme, in which these two points will be given.

5. In a rowing boat of 8 or 10 oars, and in moderate weather, go as near to some remarkable projecting point, or promontory, as you can with safety; there anchor your boat, and with an amplitude compass, constructed and balanced so as to be as little affected by the motion of the boat as possible, take the bearing of the two stasimetric objects; draw these out from their cor-

responding points on the scheme, in the opposite direction, and their intersection will be the place of the boat. Take the bearing of some remarkable part of the point, or promontory next you; judge its distance as near as you can; draw that bearing on the paper from the point found for the boat's place; and on it mark the estimated distance of the observed part of the point or promontory; which will be one point in the coast to be surveyed, and sufficiently exact for the purposes of navigation; the error being only what may arise from estimating a distance that is not above a quarter or half a mile off.

6. In the boat, take the bearing of the next point along the coast, from the point or promontory determined; draw it on the paper; mark its estimated distance on that line; and with the point of your compasses trace on the paper the figure of the coast between these two points of land.

7. Go to the point last observed, as near as you can; take its bearing from one of the stasimetric objects, which will intersect the line of direction from the first place of anchorage in the corrected distance of the second point along the coast; correct also the figure of the coast, and pencil it; and lastly, insert remarkable objects on the land, and diversities of the coast between the points, together with the rocks observed, and soundings taken by the way. Take the bearing of the third point, mark its estimated distance, sketch the curve of the coast, and correct both when you are at that point as before; and go on in this manner, from point to point, till the whole is finished.

Though this method of surveying is less perfect than

any of the other methods, yet if the card of the compass can be made to stand at rest in the boat, either by art, or the smoothness of the sea, a draught of a coast may be made in this manner, or of bays or harbours in it, which will give a distinct notion of them, and prove a pretty good direction to shipping, either along the coast, or into these bays or harbours.

Case 2. If the coast extends eastward and westward, then two stasimetric objects must be chosen, which lie about N. E. or S. W. from each other, and also their bearing and latitudes found, in order to find their distance, and form a stasimetric scheme for carrying on the survey in the same manner as before.

Example IV.

From a Vessel at Anchor in the Mouth of a Harbour, how to sketch the Harbour speedily, as far as can be seen.

Suppose fig. 6. in Plate III. is the harbour, and the vessel at V: take the bearing of any remarkable part D, right up the harbour; guess as near as you can its distance; draw out the bearing on paper, and by a convenient scale, mark on it that judged distance. In the same manner, mark a point for the bearing and judged distance of C: then draw between C and D on the paper, the curvature, or windings of the coast, as they appear to the eye. Next take the bearing and judged distance of the point B, and draw the irregularities of the coast from B to C, and to F. Do the other side of the harbour from A to D and to G in the same

manner, and the figure on the paper will have an apparent resemblance to the harbour, exhibit its dimensions something near the truth, and shew the true course into it, if there are no unperceived rocks or shoals in the way. Next, insert in the draught the most remarkable appearances of the coast, the houses, hills, trees, forts, &c. and such rocks or shoals as can be seen while you remain there. In sailing to, or from the harbour, the direction of the coast, on one, or both sides of the entrance; and within the harbour from B to C, and from A to E, may be corrected by the bearings.

This method of sketching a bay, or harbour, though far from being exact, will give such an idea of it as may be of service on some occasions; and therefore, should not be neglected when there is no opportunity of doing it more exactly.

Example V.

How to make a serviceable draught of a Harbour by a compass, or magnetic needle, only.

Plate II. Fig. 1.

Go to some eminence where the whole, or a great part of the harbour may be seen, and select two conspicuous sharp objects, as tapering hills, steeples, chimney-heads, &c. as A and C, at a competent distance from each other, and conveniently situated for a base-line; (if no such sharp objects are to be found, put up signals on the hills, or eminences;) take their bearing in degrees, and parts of a degree by the needle, or compass,

which should be provided with sights, and graduated for that purpose; and estimate by the eye the distance of C from A, as near as you can judge. At A and C take the bearing of B, and of any other remarkable object D, on the other side of the harbour; and from these data protract a stasimetric scheme on paper, wherein the points A, B, C, D, will have the same magnetic directions and proportional distances as the objects they represent.

With this scheme go to any adjacent point of land in the harbour, as X; there set down your compass, or needle; take the magnetic directions of two of the stasimetric objects that are most advantageously situated; draw them out from their respective points on the scheme, and their intersection will be the point of station X. At X take the bearing of the next point Y; guess its distance from you; take that distance from the scale; mark it on the bearing-line, and trace on the paper with the point of your compasses the curve of the coast between X and Y; take also the bearing of a and b, and of such rocks and shoals as may be seen along the coast, and draw them on the paper. Then go to Y; there take the bearing of A, or of any other of the stasimetric objects that may make a more direct intersection; draw out its direction, and its intersection with X Y will be the corrected place of Y. By the bearing of b and of a, correct likewise their positions; then the curve of the coast between X and Y may be traced with black-lead more exactly, and remarkable houses, trees, &c. inserted, and the diversities of the coast between X and Y marked. Proceed in the same manner from point to point till the whole coast-line of the harbour is delineated. If any more stasimetric objects become

necessary, let their places be determined by magnetic intersections drawn from two of the former objects.

Next, by intersections of the needle taken on land, or of the compass on sea, find the places and extent of the several rocks, shoals, and sand-banks in and near the harbour, take marks for avoiding them, and sound the depth of the water round them, and in other parts of the harbour; and mark the best anchorage.

Such a draught as this, of a bay or harbour, neither requires much skill nor time to execute; and though not exact in the distances, exhibits a just representation of the place; and will be found sufficient for directing vessels in and out without a pilot, if the channel is not *very* intricate.

Example VI.

How to sketch a Coast in sailing along it.

Plate V.

1. Prepare a sheet of large paper, with parallel black-lead lines and perpendiculars crossing them all over, as in a stasimetric scheme. Prepare also, in a book, an *Observation Table*, in which is to be wrote distinctly and regularly, the several celestial observations, bearings, distances measured by the log-line, rocks, shoals, soundings, overfalls, races of tide, and other remarks that may be made along the coast. The table may consist of seven or eight columns, disposed in the following order.

OBSERVATIONS IN NAVIGATING THE COAST OF ——

SKETCH I.

From Cape —— *to the Point* ——, *being* —— *Miles measured by the Log. The Course from Station* 1, *to* 2, *being* S $\frac{1}{4}$ W.

Year, Mon. and Day.	Mer. Alt. of the Sun.	Bearings at Station 1.	Time and Distances sailed from St. 1.		Bearings and Soundings taken at these Distances.		Bearings of Rocks, Shoals, Overfalls, and their estimated Distances, when in a Line with Points or Heads on the Coast.		Remarks on the Tides, and on the Nature and Dimensions of the Rocks, Shoals and Anchorage.
	D. M.		H. M.	Miles.		Fath.	Points and Heads.	Miles.	
1774 Jan.		A W. 17° N. a W. 22° S.	1 20 11 45	$\frac{3}{4}$ 4	A N. 5° W. b W. 20 S.	21	c, and Rock. W. 7° N. d, and Hill. S. 19° W.	$2\frac{1}{2}$	This Rock dries at low Water, and seemed 100 Yards in Length from N. E. to S. W. a Leading-mark to it is

2. Let four expert persons be appointed; one to take the bearings exactly with an amplitude compass; one to oversee the running out of the log-line, and to keep an account of the ship's way, so as to be able readily to tell the distance she has run, when required; the third to attend the heaving of the lead, to write down the soundings, and the bearings of one or two heads, points, or remarkable parts of the coast taken at each depth: the fourth, a draughtsman, to draw out the necessary bearings and distances, and to delineate the figure and windings of the coast at the several stations, and to correct their forms and dimensions while the ship is sailing along the land.

3. Begin to take the sketch, if it can be done, off some remarkable promontory, or point, that extends farther out than the rest of the coast, and may intercept and turn off the principal stream of tide, so that there may be little stream where the ship's way is to be first measured by the log. Bring the ship as near the point as can be done with safety; there drop anchor, if the depth of the water and the ground are proper for it; if they are not, make the ship *lie to,* as steadily as possible.

4. On a convenient part of the prepared paper make a *dot* to represent the place of the ship; draw a small black-lead circle round the dot to make it more conspicuous, and mark it with No. 1. for the first station. Then take the bearing of the point, or promontory you lie off; draw it out with a protractor from the dot; estimate its distance by the eye as exactly as you can; take that distance from a proper scale of equal parts, mark it on the bearing-line with a dot, and with the capital letter A. Next take the bearing of the farthest point,

or promontory, which is seen distinctly, and is intended for the limit of the first sketch; draw it out from No. 1; estimate its distance by the eye; take it from the scale, and mark it on the bearing-line with a dot and with the capital letter B. Take also the bearings, and estimate the distances, of all the points, heads, inlets of rivers or harbours, rocks, corners or heads of bays, that are seen between the two points A and B; draw them out and mark them with the small letters a, b, c, &c. as far as they are seen with any distinctness.

5. Sketch faintly (so as it may be easily rubbed out when it is to be corrected afterward) the figure of the coast between the several dots; as from A to a, from a to b, from b to c, &c. as far as 2 or 3 miles; or only as far toward B as you have a distinct sight of the bays, irregularities, and dimensions of the several parts of the coast, and let the rest be sketched as you sail past them, after their distances have been corrected by the ship's run measured by the log.

6. Look for some conspicuous tapering hill, building, steeple, or grove of trees, up the country, and several miles distant from the shore, and situated as far along the coast toward B, or beyond it, as can be found; take the bearing of that hill, or object exactly, draw it on the paper; and when another bearing of it is taken at a proper known distance from Station 1, its position will then be determined on the draught by the intersection of these two bearings, and will serve for correcting, or determining the distances of points, promontories, bays, &c. along the coast.

7. While the ship lies at Station 1, off A, let the lati-

tude be observed, if there is an opportunity, either by the sun or stars. Then get the ship under way, and sail in a straight line for the second station off B; measure the ship's *run* carefully by the log, and keep an account of the time by a watch as you sail; that when you are off any of the heads, points, or openings, marked with the small alphabetical letters, their distance may be known more exactly and corrected: correct also, by the way, the figures and dimensions of those parts of the coast which were imperfectly seen and sketched at No. 1. Also, sound the depth of the water by the way, and at each sounding take the bearing of the first point A, or of the last point B, and of one or more of the heads, or points on the coast; and likewise of the inland hill, or other object, in order to determine afterward the places of the several soundings, and to discover whether the ship is carried out of the straight course by streams of tide, currents, or lee-way, and to correct that deviation. Let such rocks, shoals, overfalls, or remarkable streams of tide as are met with, have two or three bearings taken of them when they appear in a line with any of the points, heads, or islands in the draught, and their extent estimated by the eye; and all wrote in a book, to be inserted in the draught more correctly afterward; and if there is an opportunity, to be more particularly examined another time.

8. When the ship has got to the head, or point B, the limit of the first sketch, drop anchor; or bring the vessel *to*, as at No. 1; write at the top of the *observation table* the distance between A and B, as measured by the log; and write in the table all the intermediate distances so measured; and likewise the observations, bearings, times of them, distances, soundings, and re-

marks; all of which must be reserved for future use, when the sketch undergoes another correction.

9. At station 2, take the bearing of the inland hill, or object, one direction of which was drawn out from the first station, and these two bearings will intersect in the place of the hill; which will then serve to correct the position of some parts between the first and second stations, and to determine by intersections the position of heads, points, and other parts of the coast between the second and third station; and verify, or correct, their positions measured by the log.

Take also the bearing of the top of some other conspicuous inland hill, or object, situated farther along the coast, the position of which must be determined afterward by another bearing taken at a proper distance; and *that* hill will serve for the same use between Station 3, and 4, as the other hill does between Station 2, and 3. Then take the bearing of the point, or head, B, next you; estimate its distance, and mark it on the paper in the bearing-line with a dot, and No. 2, for the second station, or beginning of the second sketch; also the bearing of a distant point or promontory, for the other end of it; estimate its distance by the eye, draw it out from No. 2, and mark it with a dot and the capital letter C. Then take bearings of all the heads, points, inlets, &c. between B and C, estimate their distances, and mark them on the bearings, and sketch the figure of the coast between the points, and proceed as was directed at Station 1, till the whole is finished in the same manner. This however is but a rough imperfect sketch, and must be corrected at more leisure,

by drawing the whole anew on clean paper from the distances and observations in the observation table.

10. If the coast lie northward and southward; observe the latitudes of the heads, points, or inlets, at every 20 or 30 miles distance, if it can be done; and by these let the principal distances be a second time corrected. If the coast lie nearly east and west, the distance cannot be corrected by latitudes, but accurately by marine chronometers.

11. When you are obliged to relinquish the work for want of day-light, or by bad weather, do it as near some head, or remarkable part of the coast as you can; take bearings, or marks, of that part, that it may be easily found when you resume the work.

12. If the coast navigated is a *strait*, or the mouth of a river, where the land on each side can be seen; at each station on one side, take the bearings of remarkable heads, points, islands, hills, &c. on the opposite side, and let that be *one* direction by which their positions and distances are determined.

A coast sketched in the above manner may contain many good harbours or bays, not perceived in sailing along it: if there be opportunity, the ship should go into some of them, until a delineation of them is made, and the entrance sounded in, and out, which would render the sketch of the coast more serviceable.

CHAP. IV.

Rocks and Shoals determined and described. Tides and Soundings. Tides described. Harbours described. Copying and reducing Draughts. Necessary Instruments.

SECTION I.

How to determine the distance and extent of Rocks, Sand-banks, and Shoals, lying off a Coast that has been surveyed; and how to avoid them by the approximating angle.

TO determine their distance, anchor a vessel at one end of the rock, sand-bank, or shoal; and in the vessel, with *Hadley*'s sextant, take the angles between three objects on shore, whose distances from one another are known, and (by Longim. Prob. III.) find the distance of the vessel from these objects, which will give that end of the rock or shoal.

Or, anchor a boat at one end of the rock, or shoal, and take the angles on shore from two stations whose distance is known, and (by Longim. Prob. I.) find the distance of the boat.

To find the extent of the rock, sand-bank, or shoal; if its extent is small, it may be estimated by the eye sufficiently near: if large, the other extremity must be determined in the same manner, by the angles taken on shore; or rather, by one angle taken on shore, and that direction intersected by a magnetic bearing at the other end before determined.

When a coast has been first carefully surveyed, the distance of rocks, shoals, and sand-banks, may be found with sufficient exactness by two bearings of some distinct part of them taken on shore with a good magnetic needle; provided the stations are not at such a distance, or position from each other, as to make too oblique an intersection.

If the shoal lies far off the coast eastward or westward, that the land cannot be seen from it; in moderate weather, anchor a vessel at the shoal, and two boats with their sails loose at a proper distance and position from each other, between the shoal and land, and so as the boats may be seen from the shoal, and the land seen from the boats. Let an observer in each boat take the angles between three known objects on land with *Hadley's* sextant; and another observer in the vessel at the shoal, take the bearing of the two boats as exactly as possible with a compass. By the first angles the distance of the boats from the land, and from each other, will be found; and by the bearings, the distance of the shoal from the boats, and of consequence from the land, will likewise be determined.

The distance of the shoal from any one of the boats may likewise be measured by the velocity of sound, by appointing two or three guns to be fired from the vessel when a signal is made, as directed in Longim. Oper. III.

If boats, or small vessels, cannot anchor in sight of the land, and, at the same time, be seen at the shoal; then sail from the shoal towards the land with a leading wind, measure the ship's run carefully with a log-line, and when the land is seen distinctly, there drop anchor; and find your distance from it by *Hadley's* sextant; and the distance run from the shoal, added to the distance of the land (and reduced to one direction, if necessary)

will give the distance and position of the shoal from the land nearly. If the shoal lie north or south from the coast, its distance may be found by the latitude, and the ship's course in sailing towards the land.

At every rock, shoal, and sand-bank, marks ought to be taken on the land for avoiding them. In doing this, chuse such objects only as are conspicuous, which a stranger may readily distinguish; and not such as are small, and insignificant, which can only be known by a person particularly acquainted with the coast. If no distinct land-marks can be found for directing vessels past the dangers, they may be avoided by the *approximating angle*, taken in the following manner. On the outside of such rock or shoal, with a sextant observe carefully the angle which two remarkable objects on the land, at a proper distance and situation, subtend; take also their bearings by the compass; and write all down for future use. And while at any other time, in sailing towards that shoal, the angle between these two objects is found *less* than the approximating angle, you are certainly without the shoal; and also without all rocks or shoals lying within a circle that may be supposed to pass through the two objects on land, and the point where the approximating angle was taken. For all angles *in* the periphery of that circle, which the two objects subtend, are equal; the angles *within* the periphery, greater than these; and the angles *without* it, less. Euc. iii. 21. The bearings will shew when you are at a moderate distance from it on either side.

To discover shoals near to any coast. In blowing weather, go to the top of a hill, cliff, or eminence, and observe where the sea appears rougher, or the waves

break more than in other parts, for there, the water is shoal; except the roughness arises from a stream of tide running against the wind; or from two streams meeting each other. Take land-marks, or the bearing of the breakers, from one, or two, remarkable places, and that will direct you to the shoal in a boat afterwards, when the weather is moderate.

Or, in a calm, observe just after slack-water, where a grey stream, or ripple on the surface, first begins; and at the smooth edge of that ripple, the shoal water will be found.

Breakers are often seen in calm weather, before a gale of wind, where the water is more shoal than in the neighbouring parts; though there may be a sufficient depth where the breakers rise.

Section II.

Circumstances to be taken notice of in describing Rocks, Shoals, and Sand-banks.

It may not be unnecessary, first to caution a surveyor against taking his accounts of rocks, shoals, tides, or harbours, solely from the information of others, without examining them himself: for few are sufficiently careful and circumstantial in their observations; and some who will affirm for fact, as if grounded on actual observation, what is only a vague opinion long entertained on uncertain authority. Even professed pilots are often satisfied with knowing how to avoid dangers by keeping at a sufficient distance from them, or in the common channel, without troubling themselves about farther particulars. Information should be made use of, but only

as a direction for farther inquiry, and to facilitate your own search and examination.

In describing rocks, banks, and shoals. 1. Take notice how far they lie from some remarkable head, or part of the nearest land; and how they bear from that, and from some other conspicuous part of the coast. 2. How far they extend in length, and in what direction. 3. What land-marks lead directly on them. 4. At what time of the tide they begin to dry, or are covered. 5. The least water on the shoal. 6. How they are to be avoided on each side. 7. The approximating angle, if distinct land-marks are not to be found.

Section III.

Affection of Tides relative to the Soundings.

As the perpendicular rise of spring and neap tides varies sensibly according to the distance and position of the moon with respect to the earth; and of the earth and moon with respect to the sun, and also by winds and weather; the depths of water proper to be inserted in draughts of the sea-coast, seem to be such only as are taken at low water in ordinary spring-tides; and not such as are found when the tides are affected by extraordinary causes. Ordinary spring-tides in most places, may be reckoned, such as happen in moderate weather, on the second or third day after new or full moon, when she is at her mean distance from the earth, and the sun not near the equinoxes. It would be extremely tedious to wait for low water spring-tide, to take all the necessary soundings. Therefore, that the depths proper to be marked may be known, whatever time they are taken,

let the perpendicular rise, or fall, of the water, in moderate weather, on the second or third day after full or change, and after quarter moon, be first found by experiment; and then the depth at low water ordinary spring-tide may be known sufficiently near, by making the following allowances at the time each depth is taken.

If the depth is taken at high-water spring-tide, then deduct the full rise of ordinary spring-tide from the depth found, and the remainder will be the depth to be inserted in the draught.

Spring-tide, at the	1st Hour before and after high-water, deduct	$\frac{11}{12}$	of the full rise of ordinary Spring-tide.
	2d Hour before and after high-water, deduct	$\frac{3}{4}$	
	3d Hour before and after high-water, deduct	$\frac{1}{2}$	
	4th Hour before and after high-water, deduct	$\frac{1}{4}$	
	5th Hour before and after high-water, deduct	$\frac{1}{12}$	
	6th Hour, or about low-water, deduct	0	

If the depth is taken at high-water neap-tide, from each depth deduct $\frac{4}{5}$ths of the full rise of spring-tide, or the whole rise except one-fifth part.

Neap-tide, at the	1st Hour before and after high-water, deduct	$\frac{4}{5}$	of the full rise of Spring-tide.
	2d Hour before and after high-water, deduct	$\frac{3}{4}$	
	3d Hour before and after high-water, deduct	$\frac{1}{2}$	
	4th Hour before and after high-water, deduct	$\frac{1}{4}$	
	5th Hour before and after high-water, deduct	$\frac{1}{5}$	
	6th Hour, or about low-water, deduct	$\frac{1}{5}$	

These allowances are not given as quite precise, for they will vary a little by several causes; but as a general rule, they will be found sufficiently exact on most occasions. Shoals and channels that have less than 4 fathoms, and are much in the way of shipping, require to be sounded at low water spring-tide, that the depth of the water there, may be accurately ascertained.

That the depths may be known exactly, be it observed, that when the moon is in the perigee, or least distance from the earth, the perpendicular rise of spring and neap-tides are about one-eight more than ordinary; and that about the equinoxes, the full rise is about one-tenth more, from that cause also: that when the moon is in the apogee or at her greatest distance from the earth, then the tides are proportionally less than ordinary: so that sometimes the greatest neap-tides may rise as high as the least spring-tides. But these and other affections of the tides have not yet been so minutely inquired into as they deserve. Hard gales of wind in any place, especially if it blow in the direction of the flood, swells the tide to an uncommon height: storms at a distance, are found sometimes to occasion an extraordinary rise of the water in places where the gale is not felt: and two streams of tide running round an island, make the tide rise higher near their meeting than it does in other neighbouring parts.

Where the depth at low water is above 5 fathoms, there a foot or two, more or less, is of no great consequence; and therefore it is sufficient if the deduction from the rise of spring tide is in proportion to the time of the tide: that is, at high water, deduct the full rise; at quarter ebb, three quarters; at half ebb, or flood, half the rise; and at three quarters ebb, or one quarter flood, deduct one quarter of the full rise from each depth found.

It is best to find the exact time of high water on the full and change days in moderate weather, when the sea is smooth; then, about 3 hours before high water, mark the place on the shore to which the tide reaches, and

write down the hour and minute as observed: on the ebb, watch till the tide fall to the same mark on the shore, and note the hour and minute; to the time between the two observations add 2 minutes for each hour, on account of the retardation of the tide, and half that sum, added to the time of the first observation, will be the true time of high water that day.

Section IV.

Circumstances to be noticed in describing Tides.

1. Observe the time of high water on the full or change day of the moon, and express it in hours and minutes.

2. Observe the perpendicular rise of the tide on the second or third day, after new or full moon; and at different seasons of the year; and when the moon is in the perigee and apogee.

3. Observe the direction of the streams of flood and ebb, near the coast, and at a distance from it; the irregularities, or variations of their direction; and at what time of the tide on the shore, the stream in the offing begins to turn.

4. Observe the greatest velocity of the stream at spring and neap-tides, off the coast, and in channels; measured by a log-line heaved from a boat or ship at anchor; or by driving with the stream, in a calm, a known distance.

5. In speaking of irregular tides, do not say, it flows 9, or 8 hours, and ebbs 3, or 4 (as is often done) when you mean only, that the stream runs 9, or 8 hours one way, and 3, or 4 the other; for that creates confusion, and sometimes mistakes. While the tide rises, and then only, it is flood; and only while it falls, is it ebb; in whatever direction these streams run. Therefore always express such an irregularity in the stream, by the time it *runs* in one direction, and the time in the other.

Section V.

Circumstances to be remarked in describing Harbours.

1. Observe if they are well sheltered, or, on what quarter they are exposed.

2. The depth of the water, and nature of the ground in the anchorage.

3. What number of ships they are capable of containing easily; and of what size, or draught of water.

4. In what part to anchor; and the land-marks, or bearings for it.

5. How the entry may be distinguished at a distance, or how to fall in with it coming from sea.

6. How to avoid rocks and shoals in the way to a harbour; or in sailing to the anchorage.

7. If there is a sufficiency of fresh water, and where it is; and what provisions the place affords.

8. In unknown places, observe what forts, or batteries are in them; what their strength is; and how near a large ship may get to them: or, in what parts of the harbour, forts, or batteries, may be advantageously erected, either for defence, or attack.

SECTION VI.

How to copy a Draught exactly.

Pin clean paper over the draught to be copied, so as they may not shift: hang, or hold, them up before a window, or the glass of a chariot, and the drawing will appear through the clean paper, and may be traced on it with a black-lead pencil, and afterwards with ink.

Or, rub the back of the draught, or as much of it as may be necessary, over with charcoal; lay the rubbed side on clean paper, and pin them together, and lay both on a table, the draught uppermost, and with a smooth-pointed tracer, of brass, ivory or steel, not very sharp, trace exactly all the coast-line first, and then other parts, and the under paper will retain the impression distinctly. If the charcoal stains other parts of the under paper, it will rub out by the crumb of loaf-bread, after the tracing is inked.

If, in the copy, you want the true meridian right up and down on the paper, instead of the magnetic: draw the true meridian first on the middle of the original

draught; then in a proper part of the clean paper (before it is pinned to the other) draw a line right up and down; and pin them together so, that the line on the clean sheet may lie exactly under, or over (according to the manner of copying) the true meridian drawn on the original; then trace the one from the other.

Section VII.

How to reduce a Draught to a smaller Scale.

A pantagraph is by far the easiest and most expeditious instrument for diminishing draughts with accuracy: but, when one of a sufficient size for large draughts cannot be got, either of the following methods will serve.

With black-lead draw the large draught all over with cross-lines, forming exact squares: draw the clean paper for the copy also over with the same number of squares; but their sides smaller, in proportion to the intended size of the scale; such as one-half, one-fourth, &c. of the length of the other: distinguish, by a stronger line, and mark with a figure, every fifth or sixth row of squares in both, so that the several corresponding squares may be more readily perceived: then, in each of the small squares, draw, by the eye, a curve similar to that which is in the corresponding larger square, till the whole is copied.

Or; draw a black-lead line from end to end of the large draught, in any direction, as A B (Plate III. Fig. 7.) by its proper scale divide that line into any number of miles, or half miles; at each division raise

perpendiculars, and draw them out to the coast-line, and mark them 1, 2, 3, &c. at each end.

On a proper part of the paper for the small draught, draw a black-lead line right up and down on the paper, for the meridian; and another line making the same angle with it, as the divided line in the large draught does with its meridian, and in the same position as a b (Fig. 8.) make a b the half, the fourth, &c. of A B, according to the intended diminution of the scale; divide it into the same number of miles, or half miles, by its proper scale, as the other is; at each division raise perpendiculars, draw them out a sufficient length, and mark them at each end, 1, 2, 3, &c. as in the other, that the corresponding perpendiculars may be readily found.

Measure on its proper scale the length of the first perpendicular on A B, in miles and parts of a mile: take the same number of miles and parts from the small scale, and set it from 1 to 1, in the small figure, and mark it with a dot: do the same in every other perpendicular, and so many points of the coast in the small figure will be found.

Then, between every two extreme dots in the small figure, sketch by the eye the corresponding curve of the large coast, and both figures will be similar in every respect. Hills, rocks, and other objects, may be drawn in the same manner.

There are proportional compasses and compasses with three legs made, by which small draughts may be diminished; but they do not answer so conveniently for very large draughts.

Section VIII.

Instruments and other necessaries for taking a Survey of the Sea-coast.

A good astronomical quadrant 14 inches radius of *Bird*'s construction; with a pirn of fine silver-wire for the plummet.

A *Hadley*'s octant, 18 inches radius.

A *Hadley*'s sextant of brass, 8 or 9 inches radius.

A bell-metal theodolite, with a spirit-level, needle, and two telescopes: the objects to be seen direct through the telescope, not inverted.

A case of good pocket-instruments.

A brass scale 3 feet long, with various lines of equal parts on it.

A wooden ruler 5 feet long.

A beam-compass of wood 2 feet long and ¾ broad, divided from the centre point along a chamfered edge into inches and eights of an inch.

A *Gunter*'s scale 2 feet long; and one a foot long.

A brass circular protractor 12 inches diameter, with an index and pricker to mark the degrees and minutes

on the paper; the graduated edge silvered. Another 5 inches diameter; and another semi-circular, 3 inches diameter.

A measuring pole 30 feet long, divided into feet; and consisting of two pieces that may be joined together endways, and taken asunder at pleasure.

A straight mahogany pole, precisely 6 feet long, divided into feet and inches; the two extreme feet into decimals of an inch.

Two iron chains, each 60 feet long; each link a foot, and the thickness of a quill; with brass marks at every tenth link.

Four iron reels, with stakes and lines, such as gardeners use.

Three wooden poles, each 10 feet long, and $1\frac{2}{3}$ inches diameter; 12 poles, each 6 feet long and 1 inch diameter: all pointed with iron on a socket, so as to pierce the ground easily and stand firm in it.

Thirty cylindrical iron pins, sharpened at one end, for piercing the ground at the end of each chain, or measuring pole; each about 8 inches long and $\frac{1}{2}$ an inch diameter.

Two canvass pockets for holding these pins, with strings for tying them round the waist of the measurers.

A good amplitude compass.

A magnetic needle, with an agat cap, brass graduated circle, and a lever, in a square wooden box, with jointed sights to lie within the box when shut.

A pocket-compass, with an agat cap.

A ship's-compass, for taking bearings in the boat; the depth of the box equal to the diameter of the card, so as not to touch the glass above, nor the bottom below with any motion; and the angle of the cap a right angle.

A good portable refracting telescope, of *Doland*'s construction.

A case of leather, divided length-ways by pasteboard into two apartments, 2 feet 1 inch long, and 18 inches broad; for holding the original draught, spare paper, scales and protractors, and securing them from rain and salt-water: also a small drawing-board, the size of the case, with leather straps and buckles, for carrying both together over a man's shoulder.

A draught-cover, 2 feet long, and 18 inches broad, made of pasteboard covered with oil-cloth, like the cover of a book, for laying the original draught in, that it may slip easily in and out of the leather case without injury.

A good stop-watch, that shews seconds.

A two-foot reflecting telescope, for observing eclipses of Jupiter's satellites, or of the moon, when necessary to find the longitude.

A large drawing-table, 7 feet long, and 5 feet broad,

with a round moulding along the edge of one side and one end, to keep the paper smooth when drawn over the table to reach the top of the draught. The board, when not in use, to come asunder in two leaves, one of the leaves $1\frac{1}{2}$ inch broader than the other: when in use, to be fastened together on the under side by two hooks, one near each end; and to be supported by a frame, or by a long three-legged stool at each end; with a loose post, or pillar, under that side where the drawer commonly sits, that the board may not warp, or bend, by his weight when leaning on it: the parts of the frame, or stools, to be made and marked so as to be easily taken to pieces, or joined, occasionally.

A folio observation-book of 4 quires medium paper, ruled at the top for a running title; and for a local title, a two-inch margin on the left side of each page; and 2 columns on the right: the first half an inch, for degrees and minutes, the other one inch, for magnetic bearings.

A folio journal-book, of 3 quires medium paper, ruled at the top; with a two-inch margin, and a small column for the days of the month on the left side of each page.

A view-book, of $\frac{1}{2}$ a quire imperial paper, opening length-ways to the extent of 2 sheets, and 12 inches broad; the leaves pasted and guarded in the middle.

Memorandum-books, each of 6 sheets demy paper, with red leather covers.

China, or such other large paper, for the original

draughts; that may bear to be often folded without cutting.

Imperial paper for the draughts.

Transparent paper, for copying parts of a draught readily for occasional service.

Writing paper.

Cartridge paper, for covering up a clean draught while it is making out.

Quills, crow-quills; cake-ink, Indian-ink; hair-pencils, some of the smallest, and some of a middling size; black-lead pencils, some of which flat; charcoal, for copying a draught by tracing it on a table; pen-knives.

A survey-boat, or six-oar'd cutter, 24 feet from stem to stern, 7 feet 3 inches broad, and 2 feet 11 inches deep, with wash-boards, masts, sails, &c. and two grap-lings, a larger and a less; the larger to have a block made fast to the ring for running 40 or 50 fathoms of rope through, in order to moor the cutter off the shore or rocks, and haul her in or out at pleasure. Also an awning of painted canvass to sleep under where it is not safe landing. Such a boat will carry, of king's allowance, one week's beer, and two week's provisions (water and firing excepted) of all other kinds for eight persons; together with a tent, bed-clothes, and cooking vessels; and is fit for rowing as well as sailing, if made light.

A captain's tent and marque, with a bed and bedding to serve two persons.

A vessel about 120 tons burden, pretty broad in the beam, and full in the bows; and of such a mould as to draw little water, and to take the ground easily; with a cabin 12 feet in the floor, well lighted, and fit to hold the above-mentioned drawing-table. Her complement of men (besides the surveyor, his assistant, and two servants) to be a master, a purser, a mate, a midshipman, a carpenter, a sail-maker, a boatswain, a cook, fourteen able seamen before the mast, and a pilot for the coast. A larger vessel cannot go into small creeks, which will often retard the service, and be found very inconvenient on a survey: a vessel much less, cannot keep sea well, nor carry provisions and other stores sufficient for the season; which is likewise a disadvantage. In hot sickly climates, and countries where seamen cannot be got easily, a greater number of hands will be necessary, and likewise a surgeon. If the surveyor has two assistants under his direction, that both may be employed at the same time, another six-oar'd cutter, with six hands more, will be necessary: and, if the coast has many shoals, sand-banks, or bar-harbours along it, or few harbours that are accessible at all times of the tide, a pilot for each boat, who is minutely acquainted with the coast, will also be necessary, to guard against misfortunes, and for finding out the shoals and channels more readily.

CHAP. V.

Sec. 1. To find the longitude by an Eclipse of one of Jupiter's satellites. 2. To continue a Meridian through a Kingdom. 3. To trace a Parallel of Latitude through a Kingdom.

Section I.

How to observe an Eclipse of Jupiter's satellites, in order to find the Longitude of the Place of Observation.

THE most exact survey of a coast cannot be reckoned complete, till its longitude from some other noted place is known: and the most careful surveys, if they extend far eastward, or westward, will require to be corrected by an observation of the longitude. It is therefore a necessary part of a surveyor's business to understand how this problem is to be performed.

For determining the longitude of one place from another on land, eclipses of Jupiter's satellites, especially of the first, are found to be the most advantageous celestial appearance. If the hour, minute, and second, when any such eclipse happens is observed in two places, the difference of these times, converted into degrees, minutes, and seconds of a circle, will give the distance of their meridians, or the longitude of one place from the other. Or, if the precise time of an immersion, or emersion at one place is known by calculation, and found by observation at any other, their difference of longitude will then be known. There are tables published an-

nually, in the nautical almanack, of the time at Greenwich, when Jupiter's satellites will be eclipsed: if therefore the time in another place, when such eclipse happens, especially of the first satellite, is found by observation, their difference of longitude will be then known.

To find the precise time of the day at which an immersion, or emersion of a satellite is observed, four principal operations are necessary. 1. *To find an exact meridian-line,* so that by it a clock, or watch, may be set either to apparent or to mean time at the place of observation. 2. *To regulate the clock or watch,* so as they may keep equal time: or, to find the error of their going, that proper allowance may be made for it. 3. *To set the clock, or watch, to mean or equal time by the meridian-line.* 4. *To observe, and note, the precise moment when the eclipse begins, or ends.*

1. *To find an exact meridian-line.* For doing this, see several ways by the stars (Part I. Chap. V.) which are more to be depended on than any observations of the sun, if the observer is not provided with the best instruments and a convenient observatory.

2. *To regulate a clock, or watch, so that they may keep equal time;* or to find how much they go fast or slow, that allowance may be made for it. The most simple and certain test of equal time is, the diurnal rotation of the earth on its axis: or, which results from it, the apparent daily revolution of the fixt stars round the axis of the earth. The clock or watch may also be regulated by equal altitudes of the sun, or by single altitudes of the sun or stars, as pointed out in the dif-

ferent treatises on nautical astronomy; but not so correctly as by observing transits of the stars, or by transits of the sun, if the transit instrument is perfectly adjusted to the polar meridian.

3. When the clock, or watch, is regulated to go equally, or the error of their going is found; *then let them be set to the mean time* of the day by the meridian-line,* in the following manner.

In a room with a large door, window, or opening towards the north, from whence your meridian-line northward may be seen (if the meridian has been marked toward the north) and another door, window, or opening towards the south, right opposite to the other, through which the sun may be seen at mid-day; hang up two plumb-lines exactly in the meridian, as far asunder as conveniently may be; let the plummets, or weights, be as heavy as the lines will easily bear; and make them swing in water, to prevent shaking by the motion of the air. Stop the clock, or watch, precisely so many minutes and seconds before, or after twelve, as are equal to the sun's equation that day, which may be found in equation tables; and with a smoked or tinged glass before your eye, observe when the centre of the sun

* Mean, or equal time is, the time shewn by a clock that is set right, and goes exactly equal. Or, it is the time shewn by the sun, corrected by the equation; or when allowance is made for the inequality of the sun's motion.

Apparent time is, the time shewn by the sun (whether in the meridian or not) without making any allowance for the inequality of the sun's motion: this is the time shewn by a good sun-dial; or found by an azimuth or amplitude of the sun.

Equation of time is, the acceleration, or retardation, of the sun's motion; or how much the sun is too fast, or too slow, of equal time.

comes exactly in a line with the two plumb-lines, and that moment call out, *now;* that your assistant may set the clock, or watch, a-going immediately. Or, which perhaps is more accurate, stop the clock as many minutes and seconds before the time pointed out by the equation, as the centre of the sun takes to move the length of his semi-diameter that day (which may be found in the nautical almanack) and when the western edge of the sun just touches the plumb-line, set the clock, or watch, a-going, and they will then be set to mean or true time. In most places it will be necessary to have a small temporary room, for an observatory, built on purpose; with an opening toward the north, and another toward the south, that may be shut at pleasure, to keep out any draught of air which would disturb the plumb-lines.

4. *To observe, and note, the precise time of the immersion, or emersion, of a satellite:* two or three days preceding the night of the eclipse, set your clock, or watch, to mean time by the sun and the meridian; and, if it can be done, examine its accuracy, the day of the eclipse also; but without altering the clock, though it has not gone quite exact; for clocks and watches often go irregular a little after they have been set back or forward: only, note how much it is less, or more than the mean time each day, that proper allowance may be made in settling the true time of the eclipse. Guess as near as you can to your longitude from Greenwich, and from thence find at what time the eclipse may be expected with you: if you are eastward of Greenwich, the nominal time it will happen with you will be so much later than at Greenwich; if you are westward of Greenwich, the time with you will be so much sooner, accord-

ing to the longitude. An hour before it is expected, adjust the telescope* to your sight, so as to see Jupiter's belts distinctly. A quarter of an hour before the eclipse is expected (or more, if your longitude is very uncertain, or if it is any satellite but the first) sit down to your telescope; and if it is an emersion, keep looking towards that part of Jupiter's disk on which you find, by the nautical almanack, it is to happen. But observe, that if your telescope inverts objects, the eclipse will happen on that part of Jupiter's disk which is diametrically opposite to that in the almanack. While you are watching the appearance of the satellite, let your assistant count every second audibly, by the beats of the clock, or by the motion of the second-hand of the watch; and the instant the satellite appears, if an emersion; or the instant it disappears, if an immersion, call out, *stop:* and immediately, the assistant should repeat the last second he counted, which the observer should note down directly, while the assistant looks to the minute, and notes it down. Write down likewise what sort of telescope you observed with, and its length and magnifying power: for eclipses will be perceived somewhat sooner by a good telescope than by one of a worse kind.

An experienced observer may count the seconds by the beats of the clock, without an assistant; but one unaccustomed to these observations, by attending to the

* The telescopes proper for observing the eclipses of Jupiter's satellites are, common refracting telescopes, from 15 to 20 feet; reflecting telescopes of 18 inches or 2 feet; and telescopes of Mr. *Dollond's* construction, with two object-glasses, from 5 to 10 feet: or, which are still more convenient, those of 3½ feet, which he has constructed with 3 object-glasses.

time and satellite together, will be liable to a mistake in one, or to inaccuracy in the other.

If it is the apparent time of an immersion, or emersion, that is observed, (which is found by setting the clock to twelve, when the sun's centre is on the meridian) it must be corrected by the equation answering to the time of the observed eclipse, to reduce it to mean time, to which eclipses are calculated in the almanack;* and the difference between the calculated time at Greenwich and the equated observed time of the eclipse, converted into degrees and minutes, will be the longitude from Greenwich. To convert time into degrees and minutes of a circle: for every hour allow 15 degrees; for every minute of time over the hours, 15 minutes of a degree; and for every second of time over the minutes, 15 seconds of a degree.

Emersions are visible only from the time of Jupiter's opposition to the sun, to the time of his conjunction: during which time, he rises after the sun sets. Immersions are visible only from his conjunction to his opposition; during which time, he rises before the sun sets.

To find when Jupiter is above the horizon, and to distinguish him amongst the stars; the easiest and readiest way is by a celestial globe, as follows. Find his place in the ecliptic from an ephemeris for the day proposed; also his latitude, or distance south or north of the ecliptic; mark that distance on the globe on a circle of celestial latitude passing through his place in the ecliptic. Rectify the globe for the latitude of the place you are

* In some nautical almanacks, they are calculated to apparent time.

in, and for the sun's place at noon, and set the hour index to 12; turn the globe till the hour index points out the hour proposed; lay a quadrant of altitude from the zenith over the point marked on the globe for his place, and it will shew his height above the horizon, and likewise his azimuth at that hour: allow for the variation of the magnetic needle, and you will have his bearing by the compass. By the altitude and bearing, Jupiter will be easily found among the stars, being the largest to appearance of the stars and planets, except Venus, which is a little brighter.

Though an eclipse of Jupiter's first satellite is much more to be relied on than the rest, because the irregularities of its motion are better known; yet eclipses of the other satellites ought not to be neglected when there is an opportunity of observing them: because such eclipses may be observed in some other place; by which the difference of longitude of these two places will be as exactly known as by an eclipse of the first satellite; and if the observations and telescopes are good, is more to be depended on than any calculation.

If the part of Jupiter's disk, and the distance from it, at which the emersion of a satellite will happen, are known, that seems preferable to an immersion; because the appearance of any luminous body is more instantaneous than its disappearance.

To find the longitude of a place by an eclipse of the moon; a meridian line must be first found; then a clock, or watch, regulated to keep equal time, and set to mean or true time by the sun in the meridian, as before directed. Then the precise time noted by a clock, or

watch, when the moon's eastern limb touches the earth's shadow; also the time when the shadow touches some remarkable spots in the moon's disc; and when the moon's western edge touches the shadow at its going off, when the eclipse is just over: these several times, compared with corresponding observations in other places, or with a calculation, will give the longitude between these places. But eclipses of the moon are not to be relied on nearer than to 3 or 4 minutes of time; chiefly from the difficulty of distinguishing the edge of the shadow precisely: and therefore eclipses of Jupiter's satellites are to be preferred to lunar eclipses, for determining the longitude of any place.

Section II.

How to continnè a meridian-line through a Kingdom, or large Country.

Plate III. Fig. 9.

First chuse some remarkable place from whence to begin the meridian, as a noted observatory, church, castle, hill, &c.: if it is near the middle of the kingdom, so that the meridian may be continued northward and southward from it, it is the better. Then provide a skilful and faithful assistant, with two sets of instruments, one set for yourself, and one for him; and six men, at least, to attend each. Provide also two convenient tents, one for lodging you, and your attendants and instruments, the other for him, his attendants and instruments. The tents will serve likewise for signals, and for making observations in, and should therefore be

so contrived, that a breadth, or two, of the cloth at each end, from top to bottom, may be laid open occasionally for observing the stars. The necessary instruments are, two good sextants, graduated so as to be capable of taking horizontal angles to parts of a minute; with a spirit level, and two good telescopes to each, with cross-wires in the common focus of the glasses, and a steady stand to support the instruments horizontally: a theodolite, and two good magnetic needles with sights; four plumb-lines, each six feet long, and two stands for them, one to support two together, about four feet asunder; six straight spiked poles, each twenty feet long, with black and white flags for their tops, to render them conspicuous, as they happen to bear on the sky, or on dark land; and a line drawn on one square side of each, right up and down, for a plumb-line to be applied to, in order to set them perpendicular: if the plumb-line is hung in a box, to screen it from the wind when applied, it would be an advantage. A borer, or some such instrument, for readily making a hole in hard ground for the poles to stand in firmly: in wind, these poles will need to be supported by ropes from the top, like a tent-pole, or by three spreading legs like a theodolite: twenty spiked poles, each six feet long; a measuring pole, or chain, with reels and lines on them, for measuring a straight line on the ground.

At the place where the meridian is to begin, find an exact meridian-line, by one of the most accurate methods explained in Part I. Chap. V., mark its extremity by a long pole set perpendicular in the ground, and a flag at top. Continue the direction of that part of the meridian farther, if necessary, by taking that pole, and the pole or object you began at in a line; or by setting other

upright poles in the same direction, till you come to the summit of a hill, or eminence, where there is an extensive view beyond it; there mark a point in the continued meridian, by a pin driven in the ground, and pitch your tent over it, so that the two tent-poles may stand parallel to the meridian, at about the distance of a foot from it, and not hinder your seeing any objects that are in the continuation of it. Within the tent (that your observation may not be disturbed by the wind) hang two plumb-lines on their stand; and when at rest, make them hang exactly in the meridian backward, or toward the beginning of the meridian, by cutting the object you began at, and one or more of the poles which continued the meridian. Stand next behind the other plumb-line, and, when both are at rest, observe if they cut any small remarkable part of a distant object which may be precisely known when you go to the object; if so, then is that part of the object in the meridian; which may be continued from it as before, by taking it and the middle of the tent in a line, till you come to some neighbouring hill, or eminence, convenient for pitching the tent on, and making observations. But if the plumb-lines do not cut such a small distinguishable part of an object (which indeed will very seldom happen) then proceed in the following manner.

Find out some hill, or eminence, on one side of the direction of the plumb-lines (the nearer to it the better) and so far off as that your assistant, at that eminence, may see your tent, and you may see his, when it is pitched on it: if they can just be perceived by the naked eye, it is sufficient; for then they will be distinctly seen through the telescopes on the sextants. Let your assistant take the bearing of the hill, and of the direction of

the plumb-lines with the magnetic needle, and note them down, and these will serve to direct him nearly to the hill and meridian: send your assistant to the hill proposed, with his tent, servants, and instruments, and let him pitch his tent on a convenient part of the hill, or the neighbourhood of it, where he can see your tent, and you his; and, at the door of his tent, set up a long pole, with a conspicuous flag at the top of it, for a signal to be observed through your telescope: set you up such another signal in the direction of the meridian, near the door of your tent, to be observed by him. As soon as his signal is up, set your sextant level, with the centre of it at the place of the signal, and take the angle which his signal makes with the meridian, or direction of the plumb-lines: write it down, and send a man with it to the assistant; when you have done, set up your signal in its place, with another flag above the former, to signify that you have finished your observation; and let them remain so till your assistant's tent is taken down.

When the assistant has set up his signal, then he must go as near the direction of the meridian as he can, by its magnetic bearing which was taken at the principal's tent, and, in that neighbourhood, find out some eminence, or convenient place, for the next station, where his own tent and the principal's can be seen: there let him set up a large pole and flag, and two, or more, smaller, in a straight line between it and the signal at his tent. Then, at his tent-signal, let him take the angle with the sextant, formed by the line of poles crossing the meridian and the direction of the principal's signal, and write that down. This angle, and that sent by the principal, added together, and subtracted from 180° will

leave the third angle of the triangle. Let him keep in the direction of the line of poles, till he finds, by the magnetic bearing that he is in, or near the meridian; then let him set up the sextant, and observe the angle between the two tent-signals; if it is equal to the fore-mentioned remainder, the centre of the sextant is in the continued meridian: if the angle observed is greater than that remainder, the continued meridian is farther from the assistant's tent than the centre of the instrument; if the observed angle is less, the continuation of the meridian is between the instrument and the assistant's tent, and the sextant must be shifted either one way or the other, but always in the direction of the line of poles, till this observed angle, and the two formerly observed at the tents, make exactly two right-angles; then the point directly below the centre of the sextant is to be marked by a pin driven into the ground, and that will be another point in the meridian continued from the first station. If this place is not convenient for a new station, he must continue the direction of the meridian by poles till he comes to an eminence, or place more convenient; there he is to mark a point with a pole, and setting up another perpendicular pole at a distance, in a line with the principal's signal, let him order his tent to be taken down, as a signal for the principal to take down his, and to come away with it and with all his other things. Then, over the direction of the meridian found by the assistant, the principal must pitch his tent; hang two plumb-lines within it on the meridian, and proceed as before to continue it farther.

If a town, wood, building, high wall, or other obstacle, shall happen in the way, so as to hinder the continuation of the meridian-line, as the wood X in Plate III. Fig. 9.

then, before you reach such obstacle, on a smooth plane mark off with the sextant A B a perpendicular to the meridian, and measure in that line as great a length as will extend beyond the obstacle, so that the perpendicular B C raised on the end of the measurement may fall on one side of the obstacle; and continue B C, if necessary, with upright poles, till you find a smooth plane extending perpendicularly cross the direction of the meridian; on that plane make C D perpendicular to B C, and measure from C to D a distance equal to A B, and D will be a point in the meridian continued from A. Then a perpendicular D E, on the line C D, will be a continuation of the original meridian: by upright poles placed in these two points, and other poles in their direction, it may be continued farther at pleasure.

If A B, the perpendicular, cannot be measured exactly, because of the unevenness of the ground, mark a line in another direction, where the ground is level, as A F, or A G, and take the angle M A F, or M A G; at F, make the angle A F C = M A F; and F C will be parallel to M A; or at G, if that is the direction chosen, make the angle A G C = M A G, and G C will be parallel to M A. Then in the right-angled triangle A B F, or A B G, having the hypothenuse A F, or A G; and, in the first, the angle A F B, equal to the supplement of A F C, or, in the other, the angle A G B = M A G, thence, by protraction, or calculation, find the perpendicular A B, and note it down. At C mark C D a perpendicular to C B; on it measure the length of A B, and it will terminate in D, a point in the meridian M A continued.

If a perpendicular C D cannot be measured exactly,

then mark out a line on either side of it where the ground is level, as C H, or C I; take the angle B C H, or B C I, according to the direction assumed, and from thence, having the length of the perpendicular C D = A B, and the angle D C H = B C H — 90°, or the angle D C I = 90 — B C I, the hypothenuse C H, or C I, may be found: measure that on the ground, and it will terminate in H, or I, a point in the meridian M A continued. At H, on C H, make the angle C H E = M A F; or, at the point I, the angle C I E = M A G, and E will be another point in the meridian M A continued. Place perpendicular poles in these two points, and by them continue the meridian to some eminence where a distant view foreward may be had; there pitch your tent over the meridian-line, hang up the plumb lines in it, and examine your meridian by the stars before you proceed farther, in case the measuring should have occasioned any inaccuracy; correct the error, if there is any, and then proceed as before, by taking the magnetic bearing of the meridian, and of some distant eminence on one side of it, and sending your assistant to pitch his tent on that eminence, in order to make the necessary observations there, and find out the direction of the meridian; and continue the same process till you reach the end of the territory proposed.

The foundation of the foregoing operations, in getting over obstructions, is so obvious from the properties of parallel lines, that it needs no explanation to one who is acquainted with the first book of Euclid's Elements. It may, however, be proper to observe, that, if the measurements are not made on smooth planes, with great care, and the angles taken and set off on the ground with great exactness, it may make a sensible deviation

in the meridian: and that therefore, after such an operation, it will generally be expedient to find the meridian anew by the stars, and proceed to continue the direction of it as before. But whether such obstructions are met with or not, it will be prudent, every 80, or 100 miles, to examine the meridian by the stars, that if any small inaccuracy has arisen, it may not affect the subsequent parts of it.

Section III.

How to trace a Parallel of Latitude through a Kingdom.

Provide a good portable instrument for taking the meridian altitude of the sun, or a star, to parts of a minute. A sextant of three feet radius, with a convenient stand to support it, may be sufficient.

At the place whose parallel is to be traced, find the latitude, and an exact meridian-line: set off, by poles placed upright in the ground, a perpendicular to the meridian, and continue that line about one degree of longitude, (suppose 30 or 40 miles) till you come to a level plane extending about one mile northward, or toward the elevated pole; there find the latitude and a meridian-line: on that meridian-line continued through the plane, measure a distance equal to the difference of the first and last latitude (which will be less than one mile) and it will terminate in a point of the parallel required. Another point in it may be marked about the middle of the perpendicular to the first meridian: for

the tangent of half a degree falls very little without the arc.

At the first-mentioned point in the parallel, mark off with poles a perpendicular to the meridian as before; continue it 30 or 40 miles forward till you find another level plane; there observe the latitude and find a meridian-line: on that meridian measure the difference of the first and last latitudes, and that measurement will terminate in another point of the parallel: and the middle of the perpendicular to the former meridian will be a fourth point. Proceed in the same manner to find other points as far eastward, or westward as is necessary, and straight lines on the ground joining these several points will mark out on the ground the parallel of latitude of the place required; or more strictly a polygon not sensibly differing from it.

The polygon will be still nearer a circle, if a perpendicular is continued east and west from each of the meridians, and two points of the parallel are marked in each of these perpendiculars, one on one side of the crossing meridian, the other on the other side, and about one-third of a degree of longitude from it.

If it happen that a smooth plane is not to be had for measuring the difference of the two latitudes along the meridian, measure a straight line in any other direction making an angle with the meridian; find that angle by the sextant, and you will then have two sides of a triangle (the difference of latitude and the measured line) and the angle comprehended between them, to find another angle, which second angle observed and marked off by a pole in the ground will determine the direction

of the third side; and where it intersects the meridian will be a point in the parallel of latitude sought.

The exactness of this operation depends chiefly on the accuracy of the instrument with which the several latitudes are taken: the sextant, therefore should be carefully adjusted immediately before each observation; and observations of stars on both sides of the zenith, ought to be prefered to those of the sun taken on one side of it.

SUPPLEMENT

TO

THE TREATISE

ON

MARINE SURVEYING.

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MARINE SURVEYING.

SECTION I.

Of an Imperfection to which Sextants are liable, not generally known to Marine Surveyors.

HADLEY'S sextants and quadrants, are all more or less liable to an aberration of the index, arising from its elasticity, and the resistance or friction in turning upon its axis; which, if not well constructed, with an easy movement of the index, the aberration will be considerable, even when the instrument is new, subject to an increase by the abrasion of the axis, after the instrument has been long in use.*

To all angles measured by sextant, must be applied the index error with its aberration, which is found, thus.

Take notice in finishing the measurement of an angle, if the index had a progressive motion foreward on the arc of the sextant, or regressive toward zero. If pro-

* This imperfection of which these instruments partake, was first noticed by the late Mr. Murdoch M'Kenzie, as mentioned in the preceding work, but has been little attended to by practitioners since his time.

gressive, and the error of the sextant is to be found by the horizon; place the index a little behind zero, with the tangent screw propel it foreward on the arc till the horizon seen direct and that seen by reflection coincide, and the quantity shewn by the index is the error applicable to angles measured with a progressive movement.

But the error may be found more exact by measuring the diameter of the sun: place the index at zero, move it foreward on the arc with the back-screw loose, till only a small part of the sun's disc appears hid by the reflected image, then tighten the back-screw, and with the tangent-screw, propel the index foreward till the edges of the disc are no longer hid, but exactly in contact, or touching each other: the quantity shewn by the index, will be the diameter of the sun measured foreward on the arc from zero, with a progressive motion of the index.

Place the index backward on the arc about forty minutes behind zero, or so as to bring the edges of the sun's disc a few minutes open, then tighten the back-screw, and with the tangent-screw propel the index foreward on the arc, till the edges of the disc are just touching each other: the quantity from zero counted backward on the arc, will be the sun's diameter measured backward from zero, but with a progressive motion of the index as before. Half the difference of these measured diameters of the sun, will be the index error applicable to all angles taken with a progressive motion of the index on the arc of the sextant.

The index error applicable to all angles taken with a retrograde motion of the index, must be found by a similar movement of the index, which is merely the converse of that stated above.

ILLUSTRATION. March 5th, 1798, at 2 A. M. with a sextant made by Ramsden, measured four angular distances of the star Regulus from the remote edge of the moon, with a retrograde motion of the index: also four distances of the star Antares from the near edge of the moon, with a progressive movement of the index.

At 7 A. M. measured the diameter of the sun on each side of zero, with a retrograde motion of the index, which gave the index error 1′ 7½″ to be subtracted from the observations of the star Regulus and the moon. Measured also, the diameter of the sun on each side of zero, with a progressive motion of the index, or by propelling it foreward with the tangent-screw, which gave the index error 1′ 42½″ to be subtracted from the observations of the star Antares and the moon, which were taken with a progressive movement of the index.

Thus, was found, a difference of 35″ index error, arising between a direct and retrograde motion of the index on the arc of the sextant, which would produce considerable inaccuracy in the longitude if wrong applied, more particularly when angular distances are measured to objects only on *one* side of the moon.

If in measuring the diameter of the sun on each side of zero, the edges of the sun's disc had been opened a little, and then drawn into contact with the tangent-screw, (which seems to be the usual practice) the index error would have been 1′ 25″, as the motion of the index would have been progressive on one side of zero, and retrograde on the other side; but the index error found thus, is applicable only to such angles as are measured both with a direct and retrograde motion of the index, and the mean of these results adopted as the measured angle, which cannot be practised unless the objects are

stationary, consequently not with angular distances of the moon.

A small aberration is also perceived in many sextants, arising from the elasticity of the index when propelled along the arc by the tangent-screw, for in finishing the measurement of an angle, the index is not *immediately* at rest, but having been *bent* a little foreward or backward by the propulsion of the tangent-screw, according as the force has been applied, it requires a little time to regain its *natural* equiform state. Thus, have I often observed after measuring an angle, and quickly viewing the vernier's scale with a magnifier, that the index was actually in motion, and continued to move several seconds of a degree after the angle was measured, before it became perfectly at rest. This aberration of the index, may be perceived in many sextants, by measuring the diameter of the sun to obtain the index error; for after the edges of the sun's disc are brought with the tangent-screw exactly to touch each other, and after a little time viewing them again to verify the observation, they will be seen either a little *open* or *shut in* with each other, according to the previous motion given to the index by the tangent-screw. And this inaccuracy is often considered to be a want of precision in the eye of the observer; whereas, it frequently results from the energy of the index to regain its equiformity.

The glasses or skreens of sextants being of different colours, admit different degrees of refrangibility of light, which also occasions some mutability in celestial observations; the fixed stars themselves being of various colours, are more or less refracted by the dispersive power of the atmosphere, according to the predominent colour of which their light is composed; those of a clear

white light being more refracted than red stars.* The light of the moon likewise being white, is more refracted than that of red stars, which would occasion a star of this colour to *appear* a few seconds within her disc when it is *really* in contact only with the edge of the moon's disc. And a similar effect, in some degree, may be perceived by viewing a red star in contact with the moon's disc, either without a skreen, or with one of a light green colour, then instantly changing it for one of a red tinge, and the star will appear separated from the edge of the moon's disc.

As metals expand by heat and contract by cold, sextants are liable by changes of temperature, to be bent or deranged from their original form,† then the error found by the index not being exactly that of the instrument in this imperfect state, hence another cause of inaccuracy in angular admeasurement. To counteract a defect of this nature, or any other appertaining to sextants; in observing lunar distances, always endeavour to obtain them from the moon to objects on both sides of her, and adopt the mean result as the longitude observed. And if these objects are equidistant from the moon, or nearly an equal number of degrees east and west of her, the index exhibiting the angles of their distance from the lunar disc, will in such case rest upon the same part of the arc of the sextant, and nearly counterbalance any imperfection of that instrument. It is therefore always advisable to observe for the longi-

* A valuable paper on this subject by Mr. Lee, Librarian to the Royal Society, is published in the Philosophical Transactions, Part 2d for 1815.

† Circles being of a more perfect form, are less liable to be de ranged, and the different indexes tend to modify or counteract an small irregularities in these instruments.

tude, by stars, or the sun and a star, nearly equi-distant from the moon, when they can be obtained, which may be seen by looking at the distances in the nautical almanack.

Section II.

Atmospherical and Terrestrial Refraction.

The images of all the celestial orbs, are deflected from their rectilineal course in passing through the atmospherical media toward the surface of the earth or sea, which deflection called also atmospherical refraction,* differs in quantity, proportionate to the humidity or density of the atmosphere, for then it is greater than in warm and dry weather, when the atmosphere is more rarefied. Atmospherical refraction is therefore fluctuating, and liable to periodical as well as to daily changes on most parts of our globe, and the mean quantity is greater in cold, than in warm climates.

In our most approved nautical works, the tables of refraction are calculated at a *mean annual rate* for the British Channel, where the refraction at the horizon is stated to be 33 minutes, or more than half a degree, that the image of a celestial orb will appear above its true place when at the horizon. But this is only an approximation to truth, as the refraction at the horizon more particularly, is subject to vary daily or hourly in

* A brief and correct description of refraction, (by my late friend, and invaluable member of the community, Dr. Maskelyne, Astronomer Royal, the great improver and supporter of nautical astronomy) will be found in the explanation of Table 1st of the tables requisite to be used with the nautical ephemeris.

the same place, and more so in places far distant from each other.

In the months of October and November 1791, lying at anchor in Surat Road, when the weather was very serene, I endeavoured to ascertain the quantity of horizontal refraction by two different methods, one of which was to compute the time of the sun's centre passing the horizon in the evening, measuring his vertical diameter and motion, and the time of the sun's disc passing the line of the visible horizon of the sea. The mean of fifteen observations taken at that time made the horizontal refraction 25′ 22″ of a degree in Surat Road, or 7′ 38″ less than the mean refraction as marked in the nautical tables for the British Channel.

When the refraction at the horizon is ascertained at any place, the refraction required for any altitude is then readily found by inspection thus. Take the refraction required for the altitude from the table of mean refraction, as many lines below that altitude, as the observed horizontal refraction is lines below the mean horizontal refraction in the table.

It has been observed above, that the mean refractions are computed for the temperature of the British Channel, or for that state of the atmosphere when the barometer is at 29,6 inches, and Fahrenheit's thermometer at 50 degrees. In order to reduce the refractions to any other state of the atmosphere, the following rule (by the celebrated Dr. Bradley) may be used when great accuracy is required.

Subtract the degrees shewn by the thermometer from 450; and add together the logarithm of the remainder, the logarithm of the height of the barometer in inches, the logarithm of the mean refraction, and the constant

logarithm 5.92665, the sum (rejecting 10 from the index) will be the logarithm of the corrected refraction.

When observations are taken near the horizon and great accuracy required, the mean refractions of the tables ought to be corrected as here pointed out, according to the actual state of the barometer and thermometer; but if the altitudes are considerable, this precision is not requisite for the usual purposes of nautical astronomy.

Terrestrial refraction is experienced contiguous to the surface of the earth, by which the images of bodies near the visible horizon are more or less changed from their true place, and the horizon itself is in a constant mutable state of elevation and depression. This results from the superincumbent atmosphere being subject to perpetual variations, according as the surface of the contiguous parts of the earth are dry, or saturated with moisture, heated by the sun's rays, or cooled by evaporation.

As the visible horizon is continually varying in height by the effects of terrestrial refraction, the altitudes of celestial bodies are always liable to inaccuracy when observed by the horizon of the sea, particularly in the vicinity of land:* it is therefore necessary to observe the

* At Table Bay, Cape of Good Hope, in Feb. 1798, I found the horizon during six days, so much disturbed by terrestrial refraction, that the latitude observed daily with two sextants, by the sun's meridian altitude in the visible horizon, differed 2 and 3 miles one day from another; nor could the rates of the chronometers be ascertained, although nearly 100 observations were taken for that purpose. In the Straits of Malacca and Sunda, great discordancy in observations for the latitude have also been experienced, when the meridian altitude of the sun was taken in the visible horizon. By observations taken with Crockatoa Peak bearing either nearly east or west, I have found a difference of 3, 4, and 5 miles in the latitude of that peak by

altitudes on shore by an artificial horizon, when the latitude of a place is to be correctly ascertained; and stars observed on both sides of the zenith, by reflection in a bason of quicksilver, are best adapted for that purpose, or the sun in quicksilver or water.

No oleaginous fluid ought to be used in observing the altitudes of the celestial bodies, for I have often found the altitude of the sun taken by reflection in different kinds of oil, to exceed about 1 minute of a degree, the altitude taken at the same time in water or quicksilver; whereas altitudes taken in these two fluids never differed from each other. Perhaps the heat of the sun's rays disturbing, or producing an ebullition on the surface of oil, may thereby derange its uniformity, rendering it unfit for observations where great precision is required.

Several mathematicians think, that bodies are refracted in some degree laterally, or obliquely, as well as vertically, and certainly the effects of terrestrial refraction are sometimes very interesting and surprising, when observed with attention. At the Cape of Good Hope and other places, when the sky was very clear, I have observed a mist or dense vapour near the horizon, which reflected a white sandy beach several miles into the sea, and gave it the appearance of a dangerous sand-bank. Vessels at a few miles distance had their images reflected upward, so that the bottom of the reflected ship seemed perched on the top-masts of the real one: another sandy beach was reflected upon the land, which gave it the aspect of barren sand-hills; every thing indeed, near the horizon, seemed vibratory and animated.

different navigators, although every possible care was taken; which discordancy may be chiefly attributed to changes of the terrestrial refraction.

Section III.

Depression of the Horizon, and the computation of elevations and distances of Terrestrial Bodies, by observing the angle they make with the Horizon.

Although tables of the *mean* refraction, and of the depression of the visible horizon, corresponding to the height of the eye of the observer above the surface of the sea, are usually to be found in nautical works, the following table may nevertheless be interesting and useful to young navigators.

Table of Elevations of Objects above the level of the Sea, and the Distance they may be seen.

Feet of Elevation.	Distance in Miles.
10	3.36
20	4.47
30	5.82
40	6.72
50	7.52
60	8.14
70	8.88
80	9.52
90	10. 8
100	10.63
120	11.65
140	12.58
160	13.45
180	14.26
200	15. 3
230	16.18
260	17.15
280	17.80
300	18.48
400	21.28

In the annexed table there is no allowance for terrestrial refraction, which in a mean state of the atmosphere may be considered about $\frac{1}{12}$ of the arc of distance.

Suppose an object elevated 200 feet above the level of the sea, just discernible with the eye elevated 30 feet.

For 200 elevation of object -	15. 3
Refraction $\frac{1}{12}$ - - - - -	1.25
For 30 feet elevation of the eye -	5.82
Refraction $\frac{1}{12}$ - - - - -	.48
The object can be seen - -	22.58 or $7\frac{1}{2}$ leagues.

If a lighthouse elevated 90 feet is erected on land 200 feet high, and just discernible from the mast-head of a ship with the eye 100 above the sea.

Land 200 feet, lighthouse 90 feet is 290 feet - -	18.14
Refraction $\frac{1}{12}$ - - - -	1.51
Mast-head of ship 100 -	10.63
Refraction $\frac{1}{12}$ - - -	.87
Discernible at the distance of	31.15 nearly $10\frac{1}{2}$ leagues.

By the admeasurements of the French mathematicians, the mean diameter of the earth is 41828814 feet, a mean

degree 365528 feet, and a mile 6092 feet. Hence, the following rule will answer to find the depression of the horizon for any given elevation.

Let it be required to find the depression or dip of the horizon for an elevation of 30 above the level of the sea.

To the diameter of the earth 41828814 feet add the elevation 30 feet, the sum 41828844 multiplied by 30 gives 1254865320, the square root of which is 35424 feet, or 5.815 geographical miles 60 to a degree of a great circle on the earth.

This operation is more readily done by logarithms. Thus,

To 41828814 + 30 = 41828844 × 30 = 1254865320 Log. 9 0986023
Feet 35424 = 4.5493021 half Log.

Required the depression of the visible horizon for an elevation of 200 feet above the sea.

To 41828814 + 200 = 41829014 + 200 = 8365802800 Log. 9.9225075
Feet 91465 = 4.9612587

And 91465 feet, divided by 6092 feet (the number stated in a mile by the mathematicians) gives 15.014 geometrical miles depression of the horizon for an elevation of 200 feet above the level of the sea. Or in other words, an observer placed with his eye at the surface of the sea, will just discern the summit of an object elevated 200 feet when it is distant 15.014 miles, or 5 leagues.

In sailing near to high mountains, when their altitudes above the sea are considerable, and the ship's course and distance correctly ascertained, their distances and elevations may be approximated, thus.

Proceeding from England towards India, May 5th, 1802. At 6 A. M., the summit of the island St. Anthony, one of the Cape Verds, bore E. $\frac{1}{4}$ N. by com-

pass, and its altitude was 3° 43′ above the horizon. Sailed S. 7° W. 19.4 miles, then it bore N. E. and the altitude was 2° 14′. Sailed S. 7° W. 10 miles, then it bore N. E. ¾ N. and its altitude was 1° 34′. Required the distance of the summit of St. Anthony from each station, and its elevation above the sea.

Geometrical Construction. PLATE I. FIG. I.

Let the centre of a circle I represent station first; from thence draw a line or rhumb E. ¼ N. for the bearing of St. Anthony; draw also a line S. 7° W. 19.4 miles, which gives Station II: on the same line continued from II. set off 10 miles, which gives Station III. From II. draw a N. E. line, from III. a N. E. ¾ N. line, these will bisect the E. ¼ N. line at A, and I A measures 18 miles, the distance of A or summit of St. Anthony at Station I: the line II. A 28.6 miles is its distance at Station II.; and III. A 37.2 miles is its distance from the ship at Station III. These respective distances may be more exactly found by trigonometry (under the case of one side and all the angles given to find the rest.)

Construct the triangles for obtaining the elevation, as follows.

Let the intersections of the rhumb lines at A represent the summit of St. Anthony, and draw from I through E (to the right hand) a line to represent the surface of the sea. On this line from the axis of the mountain A, at E, set off E I 18 miles, the distance at Station I. making ∠ I. 3° 43′. Set off E II. 28.6 miles the distance at Station II. making ∠ II. 2° 14′; and set off E. III. 37.2 miles the distance at Station III. making ∠ III. 1° 34′.

To find by computation, the height of St. Anthony above the sea, as seen from each station.

For STATION I.

Comp. ∠ I. 3° 43′ = 86° 17′	Sine	9.999086
To 18 miles or 110160 feet	Log.	5.041790
∠ I. 3° 43′	Sine	8.811726
A E at Stat. I., 7152 feet	Log.	3.854430

For STATION II.

Comp. ∠ II. 2° 14′ = 87° 46′	Sine	9.999670
To 28.6 miles or 175032 feet	Log.	5.243040
∠ II. 2° 14′	Sine	8.590721
A E at Stat. II. 6825 feet	Log.	3.834091

For STATION III.

Comp. ∠ III. 1° 34′ = 88° 26′	Sine	9.999838
To 37.2 miles or 227664 feet	Log.	5.357170
∠ III. 1° 34′	Sine	8.436800
A E at Stat. III. 6225 feet	Log.	3.794132

These results are the height of St. Anthony above the *visible* horizon of the sea by computation of the altitude observed at each station: and to find the depression of the land below the horizon at each station, observe the following rule.

Station I. is distant from St. Anthony 18 miles or 110160 feet, log. of which 5.041790 multiply by 2 gives log. 10.083580, the natural number of this 12120000000 divide by 41828814 feet the diameter of the earth, and the quotient 289 feet will be the depression of the land below the visible horizon of the sea, as seen from Station I: Station II. distant 28.6 miles, gives 732 feet depression by the same method: and Station III. distant 37.2 miles, gives 1238 feet depression of St. Anthony below the visible horizon of the sea, as viewed from those stations.

Stat. I.	Elevation of the land	7152
	Depression of do.	289
	Total elevation	feet 7441

Stat. II.	Elevation of the land	6825
	Depression of do.	732
	Total elevation	feet 7557

Stat. III.	Elevation of the land	6225
	Depression of do.	1238
	Total elevation	feet 7463

Height of St. Anthony by computation at	Stat. I. 7441	Mean.	Miles.
	II. 7557	7487 feet, or	1.22 above the sea.
	III. 7463		

Having thus computed the height of St. Anthony 7487 feet. Let the angle of depression for the whole height be computed also, or the distance that an observer's eye placed at the horizon of the sea will be from St. Anthony when its summit is close to the horizon.

To	41828814	feet, the diameter of the earth,
Add	7487	do. height of St. Anthony
	41836301	
Multiply	7487	
	292854107	
	334690408	
	167345204	
	292854107	
	313228475587	

313228475587 log. 11.4958200, half 5.7479100, the natural number of which 559600 feet depression, divide by 6120 the feet in a geometric mile, gives 91.44 miles of distance for the eye of an observer placed at the surface of the sea, to discern the summit of the island St. Anthony even with the visible horizon.

In a clear atmosphere, therefore, St. Anthony ought to be seen from a ship's deck about 30 leagues of distance, provided that the bearings of the island and measured bases were correct, upon which the foregoing

computation is founded. But as no allowance for terrestrial refraction has been made, which may be considered about $\frac{1}{12}$ of the arc of distance, and in this case would amount to 7.53 miles, corresponding to 50 feet of elevation proper to be subtracted from that of St. Anthony 7487, as computed above, leaving 7437 feet for the corrected height of the island above the level of the sea.

This method of approximating the height of mountains in sailing near to them, can only be depended on, when the ship's run is correctly known, and the altitude of the mountain considerable: it has therefore been introduced here, principally with the view of affording exercise and amusement to the young naval officer during his hours of leisure. I have nevertheless found it sometimes approach nearly to the truth; for the table mountain at the Cape of Good Hope measured 3552 feet of elevation above the sea by this method; and its height by actual admeasurement on shore, is stated to be 3585 feet.

Signal Mountain on Prince of Wales Island, by the same method was 2217 feet high, and by barometrical admeasurement 2171 feet.

The same method gave 6933 feet for the elevation of the Golden Mountain near Achen; 8688 feet for Lombock Peak on the island of this name, and 1894 feet for the height of Pulo Condore in the China sea.*

* The most approved methods of measuring the height of mountains on land, are by levelling when practicable, or by the mountain barometers fitted with professor Leslie's scale.

Section IV.

Practical remarks in surveying Coasts or Harbours.

When a coast, or the land that circumscribes a bay or harbour is greatly elevated, and such elevations correctly ascertained by the mountain barometer, or otherwise, the positions of rocks, shoals, or other dangers may be found from any of the contiguous heights, by observing the angles of their depression below the horizon with a sextant or land quadrant, and taking their bearings at the same time.

Illustration. Let N, Fig. 2, represent the base of a hill level with the surface of the sea, and E its summit elevated 400 feet, to which elevation the depression of the horizon is $21\frac{1}{2}$ miles, or the distance that the horizon H will be seen from E, and the $\angle$ at H will be $21' 30''$ or equal to the depression. Prolong the axis of the hill N E to Z, draw N H to represent the surface of the sea, and to the $\angle$ at N $90°$ add $21' 30''$ gives $\angle$ Z E H $90° 21' 30''$. A line drawn from E to H will represent the visible horizon, and if an observer at E with the sextant or quadrant adjusted, observe the rock or danger ⋕ $4° 30'$ within the horizon E H, add to this $90° 21' 30''$, will make the $\angle$ Z E ⋕ $94° 51' 30''$ corresponding to which draw the line E ⋕, and it will bisect N H the surface of the sea in ⋕ at the distance of $7\frac{3}{4}$ miles from N and $13\frac{3}{4}$ miles from H, which is the situation of the rock.

If the sand bank ◍ be observed $10°$ within the horizon, add to this $90° 21' 30''$ equal to $100° 21' 30''$ the

angle between it and the zenith, or in this case the angle Z E ● is 100° 21′ 30″; draw E ● which will bisect N H in ● at the distance of 4½ miles from N and 17 miles from H, this being its true position.

The situations of other places or dangers may be found in the same manner, if not *too far* distant from the observer, and if the station at the same time be of *great* elevation.

The angle of depression of any object below the visible horizon, might be readily observed by a Hadley's sextant or quadrant, were not the horizon frequently indistinct and obscured by haze when viewed from mountains or hills greatly elevated. A well constructed land quadrant, therefore, seems preferable for this purpose, fitted on a stand, with a spirit-level or plumb-line; for the angles of depression observed by an instrument of this kind, will be referable to the line Z N, drawn from the zenith through the axis of the hill E N perpendicularly toward the centre of the earth, and not taken from the horizon as must be necessary with Hadley's quadrant or sextant.

If they are taken with the latter, then in the triangle E N ●, for computing the distance from N to ●, are given E N the elevation of the hill 400 feet, the right angle E N ● 90°, the supplement of the observed angle Z E ● 100° 21′ 30″ = angle N E ● 79° 38′ 30″, and consequently the complement of the latter, = angle E ● N 10° 21′ 30″. Therefore, the side E N and all the angles, are previously obtained by observation, to find the side N ●.

If the angles of depression are taken with a quadrant fixed on a stand at E, (and not observed from the horizon, but by direct vision) and referable at once to the perpendicular line or axis of the hill E N; then in the right

angled triangle E N ●, for computing the distance N ●, are given E N the elevation of the hill 400 feet, the right angle at N 90°, the observed angle in this case N E ● 79° 38′ 30″, and consequently its complement E ● N 10° 21′ 30″. Or it will come under the case of trigonometry, where one side and all the angles are given to find the rest. And in the same manner, may the distance of any other object be computed, by observing with a fixed quadrant its angle from radii or a vertical line passing through the axis of the hill, or by taking its depression below the visible horizon with a Hadley's sextant. The extent and form of shoals are best distinguished from elevated situations by the discoloured water upon them, being exhibited as in a bird's-eye view of a picture, and may therefore be naturally drawn on the spot by an observer.

From places greatly elevated above the sea, this method of ascertaining the distances of proximate objects or dangers, by observing the angle of depression, has been found to answer the purpose intended; but if the objects are not viewed *far below* the line of the visible horizon, and from stations of great height, their situations cannot be correctly ascertained by this method of surveying.

The converse of this method has long been practised in measuring small distances,* viz. Taking the altitude of a ship's mast-head with a sextant when in a boat near her, which altitude is the angle at the eye, and the height of the mast being known, is an opposite side to this angle. The perpendicular height should be mea-

* My friend the late Mr. A. Dalrymple, in his Essay on Nautical Surveying, states that it was practised by the Hon. Capt. Thos. Howe, in 1759, afterward by Captains M'Cluer and Wales of the East India Company's service, and others.

sured from the truck or mast-head, to a horizontal chalk line made on the side of the ship exactly at the height of the eye of an observer sitting in a boat alongside, and in taking the altitude of the mast-head it must be brought in contact with the chalk line, which line ought to have a cross through it in a vertical direction from the mast-head. If necessary, the mast might be lengthened from 30 to 40 feet, by swaying up a light spar to any fixed mark, and securing it to the top-gallant-mast.* By this method, any distance *under half a mile* may be found correctly, if the mast of a large ship is used in taking the altitude. The true positions of shoals may be readily found in this manner, in harbours where ships are moored not far from them, by placing a boat upon the shoals, and from thence taking the altitude of a ship's mast-head, at the same time measuring the horizontal angle of the ship's mast from *two* conspicuous objects on the land, whose distance and true bearing from each other and from the ship are known; magnetic bearings are not then required, as the true place of the boat will be shewn by the meeting of the lines drawn on transparent paper according to the observed angles, when laid upon a plan of the three stations previously prepared: for when the three lines on the transparent paper are brought directly over the three stations, a pin fixed in

* If the altitude of the mast be not taken from a boat, but observed from another vessel, or from the shore, with either a sextant or theodolite, so that the height of the eye is different from the mark on the ship's side, this mark may be adjusted by a board painted black, placed erect, with its lower end touching the mark, and divided into feet, half, and quarter of a foot, that the height of the eye above the sea may correspond with some of the divisions on the board. Any change of a ship's draught of water must also be adjusted, or allowance made accordingly.

the point of meeting will be the place of observation. A station pointer may be conveniently used for this purpose, but transparent paper will answer, if a station pointer is not at hand. In a survey of Bombay Harbour, I found the method here described, very convenient in taking and protracting soundings in several places, and in ascertaining the situation and boundaries of the Middle Ground, and other shoals contiguous to shipping.

The distance of any shoal or danger near the shore, or of any point or place on the land that cannot be closely approached, may be found in the following simple manner, without using any instrument to observe the angles.

FIG. 3. Let △ B be the line of direction of the rock △, and you want to find its distance from the shore. Upon the shore at the point B draw B C perpendicular to △ B, to which give what measure you think best for the purpose intended, as 80 or 100 fathoms; and at the point C draw C D perpendicular to B C. In C D take any point E, and in the line of direction between it and △ place a stave or mark G in the line B C: measure G C and C E, and say, as G C : B G :: C E : △ B.

In surveying (as the late Mr. Dalrymple properly observes) it is always desirable to have more than *one intersection of bearings*, for if there is only one, a mistake in the bearing cannot be detected, but the coincidence of two or more intersections, proves the certainty of the determination; whereas, a disagreement, shews that there is an error either in the *position* of the *stations respectively*, or in the *bearings* from *those stations*.

Although the *length* of the *base-line*, or the *distance* between *station* and *station* is essential to be known; it is only requisite in order to determine the *scale* of the

plan: for if the *direction* of the stations from each other be known, the reciprocal positions of all places laid down by the *intersection* of *bearings* from *two stations* will be the *same*, whether the distance between the two stations be *one mile*, or ten miles; only the *scale* will be different in a *ten-fold* proportion. No person, therefore, should ever hesitate to take the *bearings* from different stations, when there is opportunity; for the measurement of a *base* may be obtained afterward.

In conformation of this statement, the late ingenious Captain Jas. Mortlock's method of surveying a bay or harbour may be introduced here; it being easily performed by any person of common ability, with great accuracy, little trouble, and without any expensive instruments.

As most of the books that treat on nautical surveying, give a numerous list of *requisite instruments*, which few young officers can procure, and are therefore discouraged from undertaking a survey, or of making plans of such places as are visited by them, for want of what they consider to be necessary instruments. Hence the following simple method obviates this *seeming* difficulty, and by giving confidence to young navigators, may probably assist in rendering nautical surveying more general.

First, make an eye-sketch of the place to be surveyed (see Fig. 4.) numbering all the points, bays, rocks, shoals, &c. Choose two stations, as A and B, from whence all the rocks, points, &c. may be seen, and so situated from each other, that the bearings of the points, &c. as taken from A and B, shall intersect at angles greater than 10°, but the nearer 90° the more accurate will be the result.

Having chosen the stations, proceed to one of them as A, and place the paper intended to receive the plan

horizontally before you, extended by pins, or otherwise, on a board securely fixed, to prevent it shifting its position while the bearings are drawing.

Stick a pin through the paper firm into the board at the part intended to represent the Station A, and lay a ruler with a straight edge on the paper, touching the pin at A and pointing towards the station B, and draw the line A B: in like manner draw lines from A towards all the points, rocks, bays, &c. numbering the lines as the points, rocks, bays, &c. are numbered in the eye-sketch. Proceed next to the Station B, and place the board horizontally before you, so that the line A B shall point back towards A, and secure the board with the same precaution as at A, to prevent it shifting: then, in the line A B, stick a pin firm through the paper into the board, in that part intended to represent the Station B; from whence draw lines pointing towards the different points, rocks, &c. as was done from A, numbering them in like manner. Now, where the lines drawn from B intersect those of the same number drawn from A, will be the place of the points, rocks, &c. to which the lines were directed from the stations. Sketch in the shore between the points, &c. and the plan is completed.

The meridian-line may be found by compass, or more correctly, by drawing the line of the sun's bearing from one of the stations, and taking his altitude at the same time. Then with the altitude, latitude, and declination, compute the azimuth, and lay it off to the right or left of the line of the sun's bearing, according as the sun was to the right or left of the meridian, and it will give the true north or south, or meridian-line.

If the distance between any two points on the shore be measured, it will give you a scale for the plan; but

it may often be found more convenient to measure off a base, as A C, from one of the stations, in a direction nearly perpendicular to the line A B; and let it be in length equal to some part of a geographic mile, as 380 feet = $\frac{1}{16}$, or 760 = $\frac{1}{8}$, or 1520 = $\frac{1}{4}$, or 3040 = $\frac{1}{2}$, or any part of a mile; then will the line A C be a scale to the plan.

It has been supposed in the foregoing remarks, that any *common board* and *ruler* would answer to illustrate the simplicity of this method of surveying: but to those provided with a *drawing frame*, it will be found convenient to extend the paper upon; and a *ruler with sights* perpendicular to its edge, will be found commodious, and require less trouble. The ease and expedition with which the whole of this method of surveying is performed, ought to induce navigators to amuse themselves in taking plans of the places they consider to be imperfectly known, which would be of general utility.

The extent of *small* shoals may be readily ascertained by dropping a grapnel on one extremity, then veering away till the boat reaches the other end, and the measurement of the rope or line used will give the extent of the shoal. Bases might also be obtained by warps, in rivers or such places as offence would be taken at open remarks.

As the magnetic needle is subject to great aberrations from various causes,* it ought to be used sparingly, and with great circumspection where accurate surveys are required, and it should not be employed in determining the direction of a fundamental base-line, without absolute necessity.

* See the Introduction to my East India Sailing Directory, where will be found a description of the aberration of the needle produced by a change of the ship's head, and from other local causes.

The most correct and expeditious method of finding the *true* direction of a base-line, is to measure with a sextant, the angle between it and the sun's limb at sun-rise or sun-set, and compute his amplitude for that time, by which, the true bearing of the sun will be known when his angular distance was measured from the base-line, and by making allowance for his semi-diameter according as the near or remote limb was used in measuring the angle, and comparing the result with the position of the base-line, the true direction of the latter will be ascertained.

The sun's azimuth may also be used for the same purpose, or to determine the true direction of any remarkable object or headland, when the altitude of the sun is *low;* but, (as the late Mr. Dalrymple well observes,) taking care to measure a *wide angle,* if the altitude of the sun be considerable; and it will always be necessary to compute the difference of azimuth of the sun and the object. If the object is not exactly in the *visible horizon,* or if the visible horizon is more or less depressed in that part, than in the part under the sun, an error will arise in finding the *difference* of *azimuth* of the *sun* and *object,* which may be considerable if the circumstances are unfavourable. But as this error is to the above-mentioned deviation of the object from the proper level, as the tangent of the angle which the great circle joining the sun and object makes with the *horizon,* the observer may easily see whether an observation is under favorable or unfavorable circumstances. When the distance of the sun from the object is 90°, *this angle* is equal to the *altitude* of the sun, but in all other distances is greater: if therefore the altitude of the sun is very great, this angle must also be very great. The less the altitude of the sun is, and the less the distance

of the sun and object differs from 90°, either in excess or defect, the less is this angle.

The usual method of taking *bearings* of land, at sun-rise, noon, and sun-set, as generally registered in nautical journals, is of very little utility in the construction of charts or plans for the improvement of navigation; although bearings of *proper* objects carefully taken under *favorable* circumstances, would essentially assist hydrographers in forming accurate charts.

For example, were navigators instead of taking promiscuous bearings, in passing through a strait or confined situation, to watch for the *transit* bearings of all the remarkable headlands, points, hills, or dangers, viz. to take the bearing of *any two* such objects when viewed in the same line, or in contact with each other, it would greatly facilitate a correct delineation of those places. And when headlands or hills on opposite sides of a strait cannot be brought into the same line of view, their transit bearing may be readily taken when the ship is in the *direct line* between them, which is easily ascertained by placing a compass with sights to it, on the binnacle or any convenient place where both objects are seen, and watching the time that they are at opposite angles from the ship, or in the same transit-line of bearing. If at the time a transit of two or more objects is under view, the sun should happen not to be many degrees above the horizon, and the angle between it and the transit-line be measured by sextant, then by computing the sun's azimuth, (or his amplitude if the time be at sun-rise or sun-set) and comparing it with the observed transit-line, the *true* direction of *this line* will be thereby found, free from *magnetic variation*, or any errors of local attraction that may arise by taking the bearings with a compass.

When a transit of two or more objects is observed, it will be useful at the same time, to take bearings of such other remarkable objects as are not too far distant, by which the angles they make with the *transit-line* will be known.

Capt. W. Owen of the Royal Navy, is of opinion, from some experiments he made during his late excellent survey of Lake Ontario, in Canada, that the relative situations of places lying nearly on the same parallel, and at *considcr'able distances* from each other, might be correctly ascertained in the night by signal rockets and chronometers, as rockets may be discerned at from 20 to 30 leagues distance, when the atmosphere is clear and the moon invisible.

Suppose two places or stations about 20 leagues distance, nearly east and west of each other, and their difference of longitude is to be ascertained by this method. First, it will be necessary to have two chronometers, which beat half seconds and perform regularly with equable rates, to be compared with each other at one of the stations, then having appointed the times for firing the rockets, an assistant must proceed to the other station with one of the chronometers, and be ready to mark the times by chronometer when the explosions of the rockets are seen according to previous appointment; which must also be done by the person who has the direction of the rockets at the original station. The time when the rockets *explode*, should be marked at both stations, and not that of their propulsion, because the former will be most perceptable at a great distance, the rocket being then at its greatest elevation. Several rockets ought to be projected, in order to prove how nearly the observations agree with each other, when afterwards compared by the chronometers, each with its

corresponding observation, and the *mean* difference of time marked by the chronometers at each station when the rockets exploded, will shew their difference of longitude.

The bearing of the stations from each other, (if not on the same parallel) might be ascertained, by having two plummets placed in the *supposed* direction of the intended projectiles, that nearest the observer to be a moveable one, in order to be readily placed in a line with the farther plummet and the rocket when the latter is seen; and this line will be the transit bearing of the two stations, by which, and the difference of longitude between them, their difference of latitude may likewise be ascertained.

Section V.

*Of Surveying by Quincunx.** Plate I. Fig. 5.

It is known that *four trees* in a *square*, with *one* in the *centre*, forms a quincunx. In like manner, suppose A to be the central vessel at anchor, and B C D E *four small vessels* at anchor.

It is proper to observe, that although five vessels will answer, in this method of surveying, nine would be more convenient; but the procedure by five, is now to be explained.

It is obvious, that from B, A and E will be in a line; from E, A and B; from C, A and D; and from D, A and C; so that B and C, and D and E, may be readily

* Chiefly from an Essay on Nautical Surveying by the late Mr. Dalrymple.

placed, as the angle at A is a right angle of 90° respectively, and each of the other angles will be half of a right angle or 45°: thus the angles A B C, A C B, A B D, A D B, A D E, A E D, A C E, A E C, are all equal to *one another*, and to *half* of the right angle at A, or each 45°, as in this case the quincunx is an *equilateral square*, its *four sides* being of equal length; and it is subdivided into four right-angled triangles.

Mr. Dalrymple recommends the form of the quincunx to be an oblong square or parallelogram, with *six equilateral* triangles contained in the circle, then each of the angles will be equal, or 60°, in all these six triangles which form the circle: see FIG. 6.

If this form of the quincunx be adopted, which may probably be found most convenient, then will the angles A B C, B A C, A C B, A D E, A E D, and D A E be each 60° and equal to one another: but the angles B A D, C A E will be each 120° or double of the above; and the angles A B D, A D B, A C E and A E C will be each 30° or altogether equal to B A D or C A E.

As vessels at anchor change their position by sheering, in order to obtain the utmost precision, it would be proper to have a light flagstaff or beacon attached by a rope to the anchor, so as to stand erect over it, and indicate nearly the spot where the anchor lies. A flag should also be displayed on board the *central vessel*, to denote the *centre* of the *quincunx*; and the distance of it to the buoy of the vessels anchor may be exactly measured by a line, so that the *station* may be in the most convenient part of the vessel, though the nearer to the foremast the better, for *its height* may be used as a *base* in small distances.

Each vessel should have at least one light skiff, or whale-boat, for sounding, furnished with a book ruled

in columns for inserting the *time* of each sounding, the *depth*, *quality* of *ground*, and the angles taken with sextant occasionally, on account of any *important change* in the *soundings*, and to correct the boats' track.

The quincunx, (when the *bottom* is not *uneven*, nor other circumstances requiring it to be limited) may be extended to any distance thought expedient; and the distance, when considerable, may be measured by *sound*. But when the *soundings* are *irregular*, or if otherwise necessary, the quincunx may be contracted, so that the *distance* may be taken by *actual measurement*, from one station to another.

When the vessels are fixed in *quincunx*, the space within it ought to be carefully *sounded*, from B to C, from C to E, from E to D, from D to B, and from A to each; and also in whatever other direction, the *nature* of the *bottom* may render expedient: for this purpose, as well as for readily prolonging, or repeating the quincunx, intermediate vessels b c d e may be very convenient.

In examining the space included in the *quincunx*, when the bottom is rocky and uneven, it may, if necessary, be *sweeped*, by a small rope, a *trauling net*,* or otherwise, to ascertain the existence of any *small rocks*, which might be missed by the *lead* in sounding.

When the space within the *quincunx* is completely examined, A moves with B, C, and stations them progressively by signal, as A 2, B 2, C 2: when this space within the new *quincunx* D, E, A 2, B 2, C 2, is carefully examined, the central vessel proceeds with D, E, and stations them in like manner, as A 3, D 2, E 2, and so progressively in the same manner.

* If this is employed, plenty of fish may probably be caught for the crew of the vessel.

The quincunx may be thus continued in any direction with facility, *upward* or *downward*, to the *right* or *left*, and to any extent, if there be anchorage, and moderate weather. But it must be observed, that if the quincunx is of the oblong form, by placing vessels at angles of 60° respectively with each other, as represented in FIG. 6. A moves with B, C, progressively, *as stated above*, if the quincunx is to be continued downward, and forms a new quincunx *below* the original one. If, however, the quincunx is not to be continued *upward* or *downward*, but *laterally*, suppose in this case to the *left*, then A must remain in its station till C moves to the left of it, at an angle of 60° with A B, and A D respectively, and C 2 will be *now* the centre of the new quincunx, which being fixed, A, E, may move together to the left at angles of 60° with C 2, B, and C 2, D; then will C 2, A 2, E 2, B, and D be the new quincunx.

The foregoing explanatory remarks are applicable to *five* vessels only; but in case *nine* vessels are employed, there will be an intermediate vessel in each side of the square, as b c d e, which will facilitate the operation of sounding; and also assist greatly the *fixing* of the *vessels* in *progression*, for then *two vessels* would always remain in the same *transit* line, to direct the *course* and *fixing* of the others, without any necessity of using a compass, which is always liable to aberrations by the motion of a small vessel or boat. The sextant, on the contrary, is as exact and commodious for use in a boat as in a ship, and therefore greatly to be prefered in boats, which are usually employed for sounding the depths in *intricate* channels, examining the extremities of shoals, and other parts that require minute exactness in their determination.

Although in describing the method of surveying by

quincunx in the preceding pages, *five vessels at least* have been considered necessary, yet there is reason to believe, that *two vessels* would answer in many places, if furnished with *three* of the *telegraph beacons,* to be *fixed* as *substitutes* for the other *three* vessels; as these beacons are moored with a light anchor and small chain, they may be transported with facility. They swim erect, well elevated above the surface of the sea, forming conspicuous marks, visible at a great distance, and consequently, well calculated for angular admeasurement.*

Section VI.

On the Magnetic Needle, Table of Distance, and to fix a Meridian-line.

A caution has already been given, against the *frequent* use of the *magnetic needle* in nautical surveying, but at times it may be more or less needful as an auxiliary. The experiments and investigation of that scientific and indefatigable navigator, the late Capt. M. Flinders, have demonstrated the influence of local attraction upon the

* My friend Capt. T. Hurd, Hydrographer to the Admiralty, being fully sensible of the value of these beacons, has recommended some of them to be attached to the surveying vessels employed under his direction. They are the invention of Mr. R. Dickenson of Great Queen Street, a gentleman of great ingenuity; and if *universally* adopted, in conformity to his plan, as published in Vol. 50, of the Philosophical Magazine, coasting navigation would be rendered less dangerous than it is at present, and probably many valuable lives and much property would be thereby saved to all the maritime countries of Europe, or wherever this excellent invention is brought into use; which no doubt, will in the course of time, be appreciated by mankind as it appears to deserve.

magnetic needle on board of different ships, particularly by the change of a ship's head in high latitudes where the variation of the compass is considerable; and its polarity is also liable to be disturbed by adventitious causes, either electrical or atmospherical. This I have several times experienced, and at two different periods in the vicinity of the Cape of Good Hope, when the sky was very serene; for in taking a series of azimuths in the morning, the variation of the compass was found to have a small progressive change with the increasing altitude of the sun. The *diurnal* variation of the needle has been often exactly ascertained by observations taken carefully at different places upon the land.

Mr. J. Garnett, an ingenious astronomer, who has resided about 22 years in America, and published several astronomical works in that country, states, that he uses the common *ring dial* for determining the variation at sea as well as on land, which shews the *true* meridian within 1° of the truth, at any time when the sun's altitude is not too great; and consequently, the variation of the compass from the *true* meridian, by holding the *ring dial* directly over the magnetic needle, and observing their difference.

Lieut. D. Ross, Marine Surveyor to the Hon. East India Company, observes in his journal of 1813, as follows.

At the foot or base of some of the high islands on the south coast of China, I have observed the needle of the theodolite to be horizontal or nearly so, when the plate of the theodolite was levelled; but when it was carried up about 800 feet high, to the summit of those islands, and there carefully levelled, with the needle pointing to the same part of the theodolite, there was observed to be a very sensible dip of the south end of the needle;

which was not only experienced by one, but by two or three different instruments. Does this proceed from the attraction of some metallic substance on the south end of the needle, (it being always longer than the north end, and would more readily be depressed,) or does the north end by being carried up so far, lose a portion of its magnetism, and therefore become lighter? If this effect is subject to a general law, proportionate to the elevation above the level of the sea, may not a *long needle* with its point traversing along a graduated vertical circle, be used to *measure heights*, after the exact dip at different elevations is known?

The following table given by Mr. Dalrymple, in his Essay on Nautical Surveying, may probably be useful; as it shews the number of *feet* and *decimal parts* of a *foot*, spread by an angle of 1°, an angle of 6′, of 3′, and of 1′, at the several distances of 1 to 10 geographic miles.* This table is not extended beyond 10 miles, because small objects on land, in this climate, will seldom be discernible at a greater distance, unless they are *extremely well defined*, in which case their angles may be taken with great exactness: but the numbers are given in *decimals*, so that the number of feet for 20 miles distance is found by looking under 2′ and taking in the first decimal, viz. 2136, for 30 miles 3204, and in this manner to 100 miles.

* Multiplying the distance in feet by 355, and dividing by 20340 will give the *number* of *feet* spread by a degree at that distance; therefore an angle of 1° at the distance of one geographic mile, admitting 6120 feet to be a geographic mile, will give in feet 106,8141592920353982300.

GEOGRAPHICAL MILES OF DISTANCE.

Angle.		1m.	2m.	3m.	4m.	5m.	6m.	7m.	8m.	9m.	10m.
1° 0′	Feet	106,81416	213,62	320,44	427,26	534,07	610,88	747,70	854,51	968,33	1068,14
6		10, 6800	21,36	32,04	42,73	53,41	64,09	74,77	85,45	96,13	106,81
3		5, 3400	10,68	16,02	21,36	26,70	32,04	37,38	42,72	48,06	53,40
1		1,78023	3,56	5,34	7,12	8,90	10,68	12,46	14,24	16,02	17,80

It may be necessary to observe, that the best terrestrial objects for angular admeasurement, are sharp terminations of peaked hills, high and prominent headlands, the spires of churches, or other conspicuous marks at a considerable distance, which can at all times be clearly distinguished. Neither a *near object*, nor a *low point*, should ever be used as a *line of observation* from a ship; for the mutability of terrestrial refraction, or the elevation and depression of the ship by the tide, or even by the *lift* of the sea, will alter the *latter*, and consequently, derange the *whole chain of angles* taken from this *fluctuating line of observation:* and the *sheering* of the ship, will have the same effect on angles deduced from the *bearing* of a *near object*.

In sailing through unfrequented seas, or in places not well explored, it is important to mark on the chart, the track passed over in the day in a different way from the night track, that they may be readily distinguished, as practised in Admiral D'Entrecasteaux's voyage.

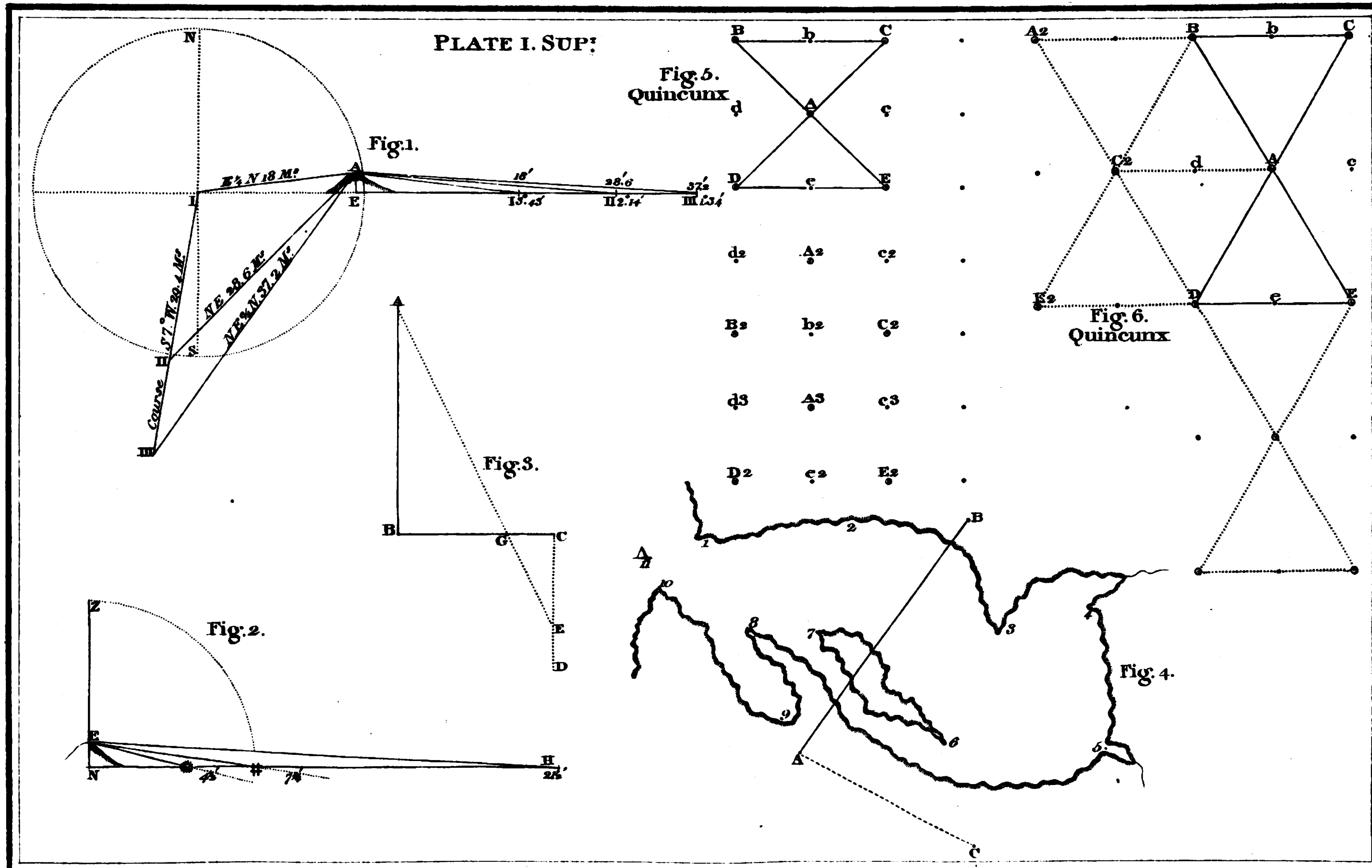
To fix a *meridian-line* in north latitude, where the pole star is visible above the horizon, the following method is simple and accurate. Compare the right ascension of the pole star with the sun's right ascension corrected for the place and time of observation; and with the sun's declination and the latitude of the place, compute the instant of time that the star will be on the meridian (either *above* or *below* the pole.)

With a clock or good chronometer, the rate of which is known, mark the precise time that the star will come on the meridian, and having two plummets ready at a considerable distance from each other, one of them moveable, and both nearly in the plane of the meridian, the instant of time shewn by the clock or chronometer that the star is on the meridian, move the sliding plummet until the star and both plummets are seen in a direct line. Then a line joining the plummets will be a *meridian-line.* Any other star situated near the pole will answer, but the bright star in the tail of the Little Bear, called the Pole Star, being nearer to the pole than any other, answers best for this purpose, on account of its slow motion in passing the meridian. The bright star in the Cross, will answer in the same manner to fix a meridian-line in the southern hemisphere.

FINIS.

Plummer and Brewis, Printers, Love Lane, Eastcheap.

PLATE I. SUP.



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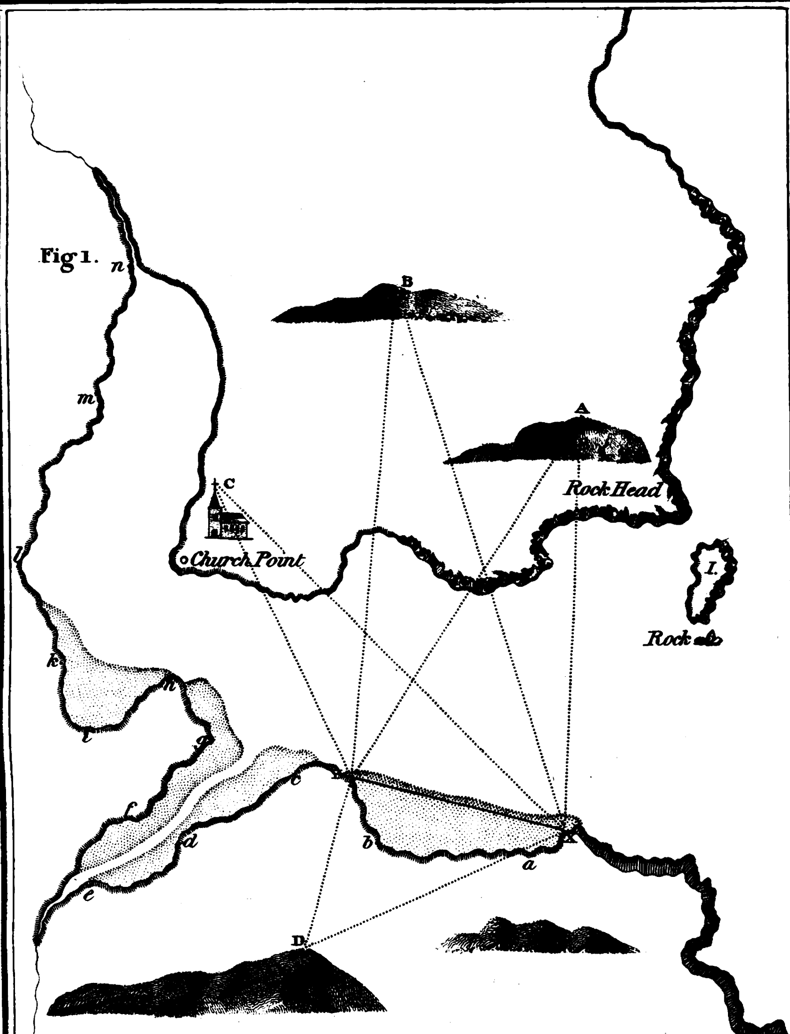
J. Walker Sculp.

PLATE II.



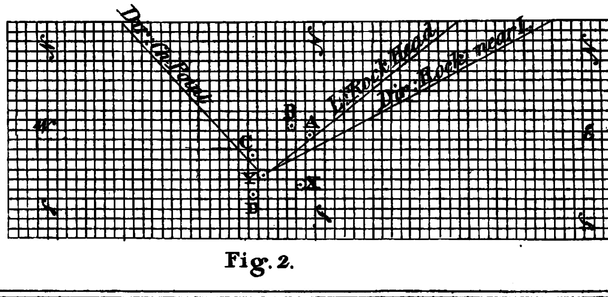


Fig. 2.

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PLATE III.

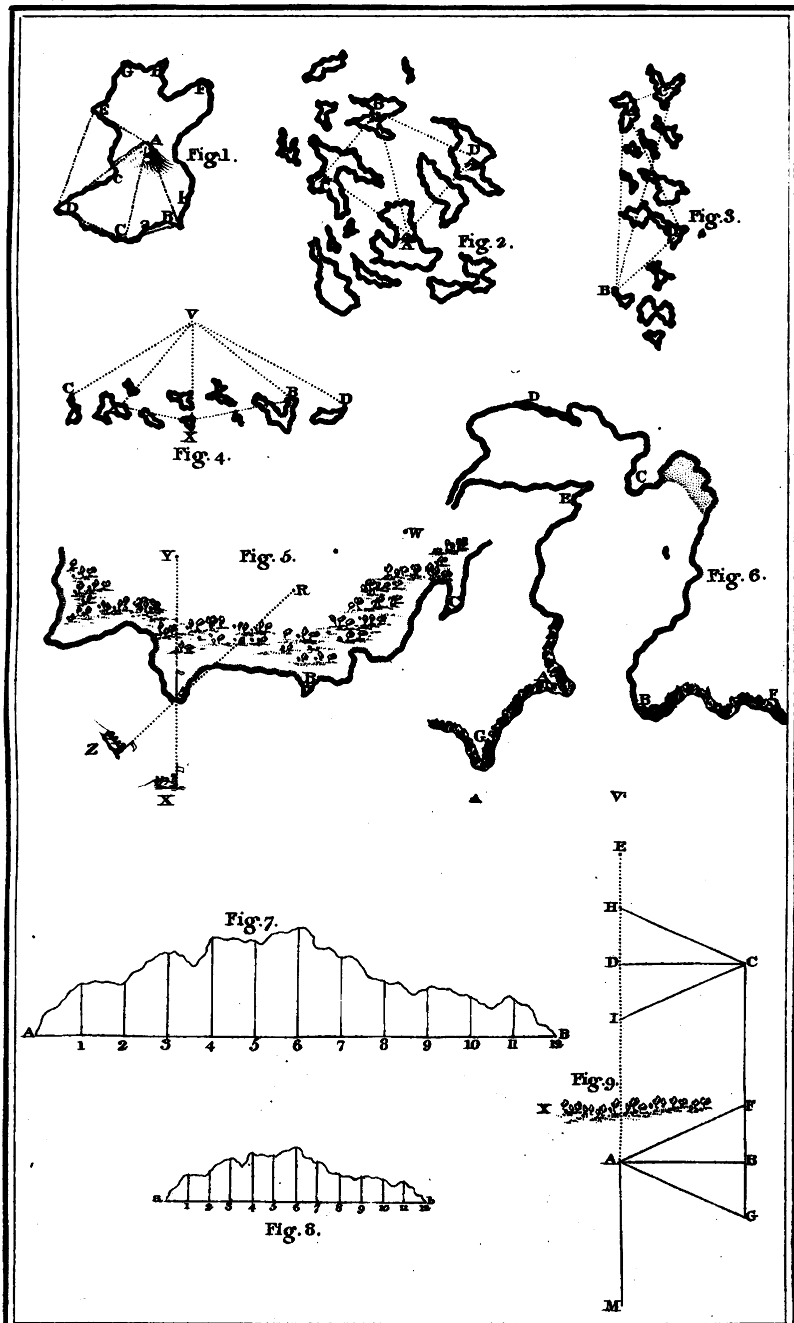

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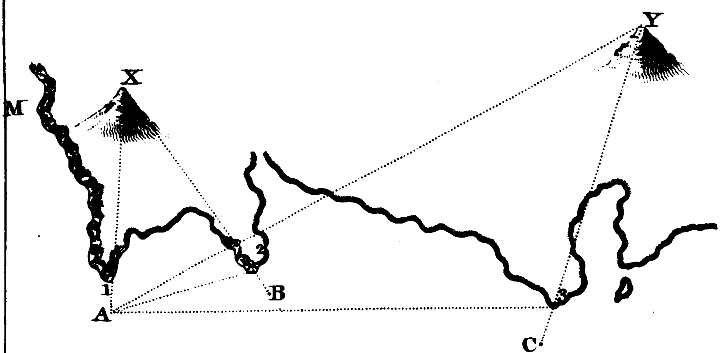
PLATE IV.

A Coast extending Northward & Southward to illustrate Example III. Chap III. PART II.



X and Y are two tapering Hills (mentioned in Article 2) whose Bearing may be taken at M, and their Distance found by their Latitudes and Variation of the Compass.

1, is the remarkable Promontory at which the Delineation begins (mentioned Article 5) 2 and 3, the second and third Points of the Coast (mentioned Art. 6 and 7)

A, B, C, are the three Stations of the Boat near these Points.

The dotted Lines are Compass-bearings taken at the several Stations, by whose intersections the Places of the Boat and Distances of the Points of Land are determined.

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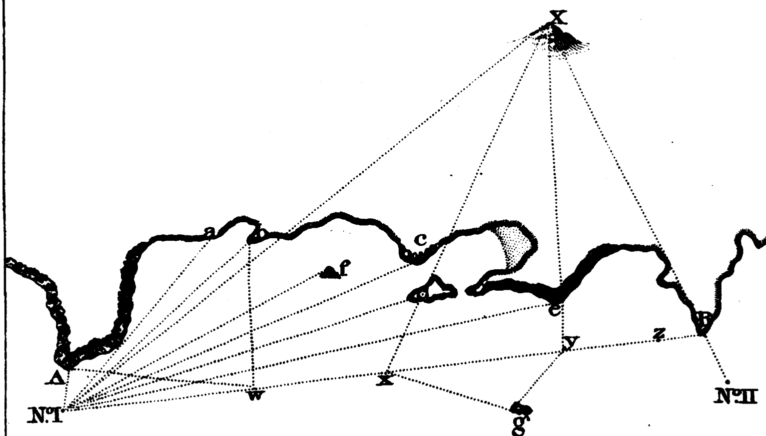
PLATE V.

A Coast to illustrate Example VI. Chap. III. PART II.



From A to B is the extent of Sketch 1.st X, an Inland Hill determined by its Bearing at N.°I, *and* N.°II, *or more advantageously at* y.

N.°I *is the first Station, or Anchorage, of the Ship; its Distance from* A *is by Estimation.*

N.°II, *the second Station, or Anchorage; its Distance from* B *estimated, or determined by two Bearings of Objects advantageously situated.*

IA, Ia, Ib, &c. *the Bearings of the several Points, Rocks &c. whose Distances from* N.°I, *are all to be marked, at first, by Estimation, and the Coast faintly sketched between them.*

IB, *the Ships Course, or the Line in which she is supposed to sail toward the second Station measured by the Log.*

w, x, y, z, *are Distances on that Line, measured by the Log; at each of which some intersecting Bearings are taken and drawn out while the Ship is under Sail, for correcting the former Estimated Distances of the Points, Rocks &c. and then sketching more exactly the Figure of the Coast between the several Places.*

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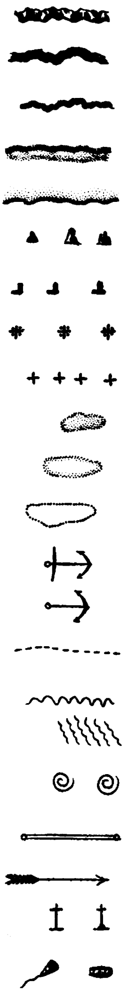
PLATE VI.

Marks *and* Diversities *to be represented in* Draughts



Rocky Cliffs

Grassy or Clay Cliffs

Low Rocky Shore

Sandy Beach, so far as dries at low water

Sandy Soil near the Shore

Rocks always above Water

Rocks that cover and uncover daily

Rocks that dry with Spring-tide only

Rocks always under Water

Sand-banks that cover and uncover daily

Sand-banks that dry with Spring-tide only

Sand-banks always under Water

The safest Anchorage

Stopping Places

The best Channel

An Eddy

Overfalls; or rough, breaking Seas

Whirlpools

Leading-marks, for avoiding Rocks and Shoals

The Direction of the Stream of Flood

Beacons, or Perches

A Can and Cask Buoy

Below the Soundings,(r) signifies rocky ground;(cl) clay;(m) mud;(g) gravel; (oz) ouze;(s) sand;(st) stones;(crl) coral;(sg) sand and gravel;(ssh) sand and shells. The numeral Letters shew the time of High Water on the full and change days of the Moon. Perpendicular rise of ordinary Spring-tide ___ Feet, Neap-tide ___ Feet.

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APPENDIX

BY CAPTAIN W. OWEN

OF THE

Royal Navy.

IN a late republication of Mr. M. Mackenzie's Treatise on Marine Surveying, by the Hydrographer to the Honorable the East India Company, Mr. Horsburgh, at page 174, a reference is made to my authority for a method* of finding the differences of longitude between places not very remote from each other, by artificial terrestrial phenomena; but as Mr. H. has in some measure mistaken me, by supposing it necessary to compare the clocks or watches together at one station, and there to obtain their respective, actual, and relative

* This method of finding the difference of meridians, noticed at page 174, was the result of a verbal communication by Captain Owen, where his meaning in some degree appears to have been misapprehended by me, from want of due consideration; for, as the velocity of light is *nearly* instantaneous, a sky-rocket or corruscation, will be observed at the same moment of *absolute time* from two or more different meridians; and consequently, if the apparent time of observation is ascertained and noted down for each station by a clock, or chronometer, and these relative times compared with each other, the difference of longitude in time, is thereby ascertained.

rates, which is altogether superfluous, and as I am of opinion that such phenomena furnish excellent means for the improvement of geography and the geodetic art, I am induced to offer the following remarks:

On the use of certain artificial phenomena, to note the same instant at two or more distinct places, for the purpose of deducing their relative longitudes from a comparion of the times at those places.

The use of celestial phenomena, for determining the relative longitudes of places on the surface of the earth is too well known to require any elucidation; for my present purpose, however, it may not be superfluous to observe, that their use is merely to note the same instant of absolute time at the places whose relative longitudes are required. The ends of geography and navigation would, therefore, be much furthered, if such phenomena happened so frequently as to render observations of them more readily, and more frequently attainable. At present the eclipses of Jupiter's satellites, and the situation of the moon relatively to the sun, or stars are the only means in use; and the want of simultaneous observations, or of perfect tables of those particulars to shew the exact time of their falling out at any particular point of the earth's surface, (Greenwich for example) added to the difficulty of observing them with accuracy, render the results of such observations for longitude but approximations to the truth, and therefore not at all available for geodetic purposes.

Many years ago, the sky rocket suggested itself to me as eminently calculated for such a purpose, within limited distances, and when a prisoner of war at Mauritius, I had an opportunity of witnessing their use as signals to a much greater distance than I had before imagined possible, and which in my opinion fully de-

monstrated the possibility of using them for geographic purposes. At Mauritius, rockets were used to warn their vessels approaching that Island and Bourbon, of the presence and situation of the British cruizers; these were commonly seen from island to island, or thirty leagues, and notwithstanding the land of both islands is extremely high, so as to be seen in clear weather more than twenty leagues; yet, it was the constant practice of the French vessels to get sight of the rockets before they made the land. It being thus demonstrated that rockets properly constructed may be seen clearly at such immense distances, it naturally occurred, that if the instant of their explosion could be marked, either by the sudden extinction of light, or by a sudden blaze or flash, that they might be used for determining differences of longitudes between places not exceeding sixty leagues asunder in favourable weather, with the probability that an error in time could not exceed a second, or not more than a hundred feet in the whole distance.

During my survey on the lakes of Canada, I caused an experiment to be made with six half pounders, or common signal rockets, to be observed from two stations thirty miles asunder; the result of which gave the differences of time by four of them, within half a second of each other; the times by the fifth differed a second and an half, and the other was not noted at one station. The result of this experiment induced me to take measures for using the rockets in the survey of the coasts of Lake Erie, where their use would have been completely exemplified, had I not been recalled as we were in the act of commencing our operations there. No doubt, however, can exist, that rockets might be so constructed as to answer such purpose perfectly within the

limit of eighty or a hundred miles, so as to obtain the differences of time within a portion of a second, particularly if aided by any mechanical means of subdividing a second into smaller portions of time.

In nautical surveying the rocket may be used with peculiar advantage in many cases, since very long bases could thus be measured astronomically with great expedition, ease and accuracy, in all situations, so as to embrace in a few observations the whole extent of operation; whereas, when geodetic operations are continued from short linear bases, very much labor and pains are requisite, as well as time, and continued accuracy of observation, to prevent the accumulation of errors, both in distance and direction; which last indeed, cannot be preserved in operations continued to any distance without the aid of astronomical observations, and abstruse calculation.

This use of the rocket, therefore, affords a ready means of tracing the extent of a country, and its most requisite features only, with great ease and accuracy without the necessity of building a system of triangles, which requires so much time and labor.

At first view, however, it may be supposed that the timekeeper, unassisted by such a device, furnishes a more ready method of performing the same thing; but unfortunately it is found in practice that such nice instruments can rarely be moved by land carriage without altering the rate, and at every station much time is necessarily required to examine it, and after all, its results are merely the results of probabilities; this reflection, however, is not meant to detract from the use

of time-keepers on ship-board, where their rates can more safely be relied on, nor for the general purposes of geography, where extreme accuracy is of less importance.

On the same principle other methods may be devised of obtaining the same end, which may in their use depend more or less on the topography of the field of operation; for example, Argand's lamps and reflectors being so placed at an intermediate point that their light may be seen from the stations whose relative longitudes are required, and a hood so suspended as to eclipse, or to shew it by an instantaneous operation; observations may be multiplied by these means to any proposed extent, by any required number of repetitions: this method by a fixed light has the advantage that it may be viewed through a fixed telescope with extreme precision, and thus appears to offer means of obtaining the measure of a degree of longitude in combination with geodetic operations with more facility and certainty than any other; on the other hand, however, it requires more preparation and arrangement than the rocket.

An example taken from the *Connaissance des temps* for 1810, will shew to what extent this method by lamps and reflectors might be used. It is observed by the mathematicians employed in measuring arcs of the meridian, that owing to the very great distance of Formentaria from the southern coast of Spain, they were obliged to have recourse to Argand's lamps and reflectors, and to night observations to connect that island by triangulation with the coast of the Continent, and that one side of one of these triangles measured one hundred and sixty thousand metres, or about twenty-

eight leagues; it is therefore possible so to arrange such lamps as to be seen at the same instant from two places fifty-six leagues asunder.

The common blue light or false fire used on shipboard for signals, offers a similar method of ascertaining difference of longitudes within more limited distances, by noting the instant of extinction, and I had at one time arranged for a set of experiments with them for this purpose, which, however, circumstances prevented from being carried into execution.

Common fires may also be similarly used in hilly countries.

And it may not be improper to suggest the *possibility* of using accidental meteoric phenomena for the same purpose.

The only article on the subject that I have seen in print is to be found in the *Traité de Geodésie*, by L. Puissant, published in quarto at Paris, 1805, at page 299, from which the following is extracted.

" An instantaneous terrestrial phenomenon which
" could be seen from two different places might be
" substituted with advantage to celestial phenomena.
" If from a very elevated situation during a clear night
" flashes of light were produced at several times by
" firing small quantities of gunpowder, and if two ob-
" servers (each furnished with a clock at the places be-
" tween which the difference of longitude is required),
" observed the instants when the flashes were seen, the
" mean of the differences of all the corresponding times

"by the two clocks regulated similarly will be the dif-
"ference of longitude required; and the time when the
"same flash was seen at both stations must be abso-
"lutely the same for places, near or distant, owing to
"the extreme celerity with which light moves. Either
"by this method or by the aid of chronometers, the re-
"lative situations of two neighbouring islands may be
"fixed, that cannot be connected by triangles."

It only remains to observe, that the number of stations are not necessarily limited to two, but that the same phenomenon may evidently be observed from an indefinite number of stations at the same instant by which *all* their relative situations might be ascertained.

It is possible, if the attention of the Board of Longitude were directed to this subject, that it might recommend some inducement to be offered to artists to perfect rockets for such observations, and to contrive mechanical means for common use, for obtaining smaller divisions of time than the second of a minute; and also, that experiments of the methods herein suggested should be tried between some convenient and known stations, to shew how far they may be practically useful for the perfection of geography.

W. F. W. OWEN.

CAMDEN TOWN,
1st. January, 1820.

10

p. 75. So lange die Wollenmütze aufgeschlagen auch
[illegible], bedarf es kein [illegible]
auf d. [illegible]. überhaupt könnte vieles
mit d. Mensch abgemacht werden.

Ebenso ist d. [illegible] [illegible]
der Methode v. Strauß, [illegible]
u. Methode d. [illegible] vielleicht auch die
Catoptr. [illegible]